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# 区间直觉梯形模糊几何 Bonferroni 平均算子及其应用

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**摘要：** 研究了决策信息为区间直觉梯形模糊数且属性间存在相互关联的多属性群决策问题，提出一种基于区间直觉梯形模糊几何加权 Bonferroni 平均算子的决策方法．基于区间梯形直觉模糊数的运算法则和几何 Bonferroni 平均算子，定义了区间直觉梯形模糊几何 Bonferroni 平均算子及其加权算子．给出了这些算子的一些性质，建立基于区间直觉梯形模糊几何加权 Bonferroni 平均算子的多属性群决策模型．最后，将该方法应用在供应商选择的群决策问题中，结果表明了该方法的有效性与可行性．

**关键词：** 区间直觉梯形模糊数；几何 Bonferroni 平均算子；多属性群决策
**中图分类号：** C 934　　**文献标志码：** A

# Interval-valued Intuitionistic Trapezoidal Fuzzy Geometric Bonferroni Means and Its Application

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**Abstract**: For solving multiple attributes group decision-making (MAGDM) problems where attribute values are in the form of interval-valued intuitionistic trapezoidal fuzzy numbers and attributes are associated with each other, an approach was proposed based on interval-valued intuitionistic trapezoidal fuzzy geometric weighted Bonferroni means (IVITFGWBM) operator. The concepts of interval-valued intuitionistic trapezoidal fuzzy numbers were introduced, and interval-valued intuitionistic trapezoidal fuzzy geometric Bonferroni means (IVITFGBM) operator and IVITFGWBM operator were defined based on operational laws and Bonferroni means operator. Meanwhile, the related properties were analysed, then a model of multi-attribute group decision making was constructed based on IVITFGWBM operator, which was for making decisions combined with sort methods. The approach was further applied in group decision-making problems of supplier selection, and the results show that the developed approach is feasible and effective.

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**Key words**：*interval-valued intuitionistic trapezoidal fuzzy number*；*geometric weighted Bonferroni means operator*；*multi-attribute group decision making*

自 Atanassov 提出直觉模糊集[1]（intuitionistic fuzzy sets，IFS）以来，因 IFS 综合考虑隶属度、非隶属度和犹豫度三方面的信息，能更加细腻地描述和刻画客观世界的模糊性本质，众多学者对 IFS 进行了深入研究．目前关于 IFS 的拓展形式主要有区间直觉模糊集（interval-valued intuitionistic fuzzy sets，IVIFS）[2-4]、三角直觉模糊数（triangular intuitionistic fuzzy numbers，TIFN）[5-6]、直觉梯形模糊数（intuitionistic trapezoidal fuzzy numbers，ITFN）和区间直觉梯形模糊数（interval-valued intuitionistic trapezoidal fuzzy numbers，IVITFN）[7]．王坚强[7]于2008年首次提出了 IVITFN 的概念，但与其相关的研究并不多见．因 IVITFN 的隶属函数和非隶属函数用区间数描述，在刻画客观世界的模糊性本质方面比 IFS，IVIFS，ITFN 和 TIFN 更为精细和准确，因而引起了众多学者的关注．万树平[8]探讨了 IVITFN 的运算法则，定义其得分函数和精确函数，给出了 IVITFN 的排序方法，定义了 IVITFN 的加权算术平均（IVITFNWAA）和加权几何平均算子（IVITFNWGA），并将其应用于多属性群决策（MAGDM）领域．Wu 等[9]研究了 IVITFN 的加权几何算子（IVITFNWG）、有序加权几何算子（IVITFNOWG）和混合几何算子（IVITFNHG），并给出 MAGDM 方法．汪新凡等[10]定义了新的 IVITFN 加法运算法则，结合 IVITFN 的期望值给出了新的 IVITFN 加权算术平均（IVITFNWAA）算子、有序加权平均（IVITFNOWA）算子和混合集成（IVITFNHA）算子．

上述研究的 IVITFN 集成算子仅考虑了属性间相互独立的情况，实际决策中，不同属性间会存在不同程度的联系，如互补、冗余、偏好关系等．有关属性间具有相互关联的 IVITFN 集成算子却不曾见到报道．基于 Bonferroni 提出的 BM 算子[11]能够很好地捕获输入变量之间的相互关联情况，其可以将多个输入变量集结为一个输入变量，是一种介于最大和最小之间的集成算子．Xu 等[12]在直觉模糊环境下拓展了 BM 算子进行 IFS 的信息集成，另外将 BM 算子与 IVIFS 结合，提出了 IVIFBM 算子，研究了相关的性质，并应用于 MAGDM 领域[13]．Dutta 等[14]将 ITFN 和 BM 算子结合，提出了直觉梯形模糊 Bonferroni（ITFBM）算子和广义直觉梯形模糊 Bonferroni（GITFBM）算子．Xia 等[15]研究了几何 Bonferroni 平均（GBM）算子，在直觉模糊环境下将 IFS 与 GBM 算子结合，提出了直觉模糊几何 Bonferroni 平均（IFGBM）算子和直觉模糊几何加权 Bonferroni 平均（IFGWBM）算子，同时研究了其性质，并通过 MADGM 算例验证了 IFGWBM 算子的有效性和实用性．

基于以上分析，本文将 IVITFN 与 GBM 算子相结合，提出 IVITFGBM 算子和 IVITFGWBM 算子，同时研究它们的一些性质．最后给出一种基于 IVITFGWBM 算子的 MAGDM 方法．

## 1 基本理论

### 1.1 IVITFN 定义

**定义 1**[7-9] 设 $\tilde{\alpha}$ 实数集上的一个 IVITFN 的区间值隶属函数为

$$\mu^{+}_{\tilde{\alpha}}(x)=\begin{cases}\dfrac{x-a}{b-a}\mu^{+}_{\tilde{\alpha}}, & a\leqslant x<b\\ \mu^{+}_{\tilde{\alpha}}, & b\leqslant x\leqslant c\\ \dfrac{d-x}{d-c}\mu^{+}_{\tilde{\alpha}}, & c<x\leqslant d\\ 0, & \text{其它}\end{cases}$$

和

$$\mu^{-}_{\tilde{\alpha}}(x)=\begin{cases}\dfrac{x-a}{b-a}\mu^{-}_{\tilde{\alpha}}, & a\leqslant x<b\\ \mu^{-}_{\tilde{\alpha}}, & b\leqslant x\leqslant c\\ \dfrac{d-x}{d-c}\mu^{-}_{\tilde{\alpha}}, & c<x\leqslant d\\ 0, & \text{其它}\end{cases}$$

其区间值非隶属函数为

$$\nu^{+}_{\tilde{\alpha}}(x)=\begin{cases}\dfrac{b-x+(x-a)\nu^{+}_{\tilde{\alpha}}}{b-a}, & a\leqslant x<b\\ \nu^{+}_{\tilde{\alpha}}, & b\leqslant x\leqslant c\\ \dfrac{x-c+(d-x)\nu^{+}_{\tilde{\alpha}}}{d-c}, & c<x\leqslant d\\ 0, & \text{其它}\end{cases}$$

和

$$\nu^{-}_{\tilde{\alpha}}(x)=\begin{cases}\dfrac{b-x+(x-a)\nu^{-}_{\tilde{\alpha}}}{b-a}, & a\leqslant x<b\\ \nu^{-}_{\tilde{\alpha}}, & b\leqslant x\leqslant c\\ \dfrac{x-c+(d-x)\nu^{-}_{\tilde{\alpha}}}{d-c}, & c<x\leqslant d\\ 0, & \text{其它}\end{cases}$$

其中，$0\leqslant\mu^{-}_{\tilde{\alpha}}\leqslant\mu^{+}_{\tilde{\alpha}}\leqslant1$；$0\leqslant\nu^{-}_{\tilde{\alpha}}\leqslant\nu^{+}_{\tilde{\alpha}}\leqslant1$；$0\leqslant\mu^{+}_{\tilde{\alpha}}+$

$\nu^{+}_{\tilde{\alpha}}\leqslant 1$；$a,b,c,d\in\mathbb{R}$，则称 $\tilde{\alpha}=([a,b,c,d];[\mu^{-}_{\tilde{\alpha}},\mu^{+}_{\tilde{\alpha}}],[\nu^{-}_{\tilde{\alpha}},\nu^{+}_{\tilde{\alpha}}])$ 为 IVITFN，当 $\mu^{-}_{\tilde{\alpha}}=\mu^{+}_{\tilde{\alpha}}$，$\nu^{-}_{\tilde{\alpha}}=\nu^{+}_{\tilde{\alpha}}$时，IVITFN 就退化为 ITFN，其几何意义如图 1所示．

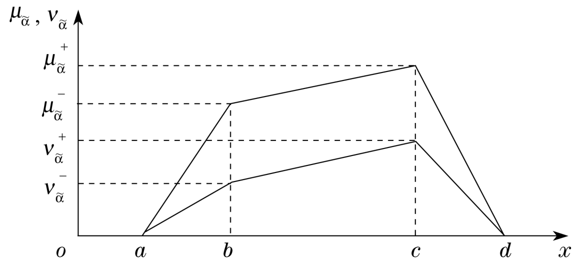


**图 1　区间直觉梯形模糊数**

**Fig.1　Interval-valued intuitionistic trapezoidal fuzzy number**

### 1.2　IVITFN 的运算法则

**定义 2**[8-9]　设 $\tilde{\alpha}_i=([a_i,b_i,c_i,d_i];[\mu^{-}_{\tilde{\alpha}_i},\mu^{+}_{\tilde{\alpha}_i}],[\nu^{-}_{\tilde{\alpha}_i},\nu^{+}_{\tilde{\alpha}_i}])(i=1,2)$为 2 个正 IVITFN，$\lambda\geqslant 0$，则有

$$\begin{aligned}\tilde{\alpha}_1\oplus\tilde{\alpha}_2=&([a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2];\\&[\mu^{-}_{\tilde{\alpha}_1}+\mu^{-}_{\tilde{\alpha}_2}-\mu^{-}_{\tilde{\alpha}_1}\mu^{-}_{\tilde{\alpha}_2},\\&\mu^{+}_{\tilde{\alpha}_1}+\mu^{+}_{\tilde{\alpha}_2}-\mu^{+}_{\tilde{\alpha}_1}\mu^{+}_{\tilde{\alpha}_2}],\\&[\nu^{-}_{\tilde{\alpha}_1}\nu^{-}_{\tilde{\alpha}_2},\nu^{+}_{\tilde{\alpha}_1}\nu^{+}_{\tilde{\alpha}_2}])\end{aligned}\tag{1}$$

$$\begin{aligned}\tilde{\alpha}_1\otimes\tilde{\alpha}_2=&([a_1a_2,b_1b_2,c_1c_2,d_1d_2];\\&[\mu^{-}_{\tilde{\alpha}_1}\mu^{-}_{\tilde{\alpha}_2},\mu^{+}_{\tilde{\alpha}_1}\mu^{+}_{\tilde{\alpha}_2}],[\nu^{-}_{\tilde{\alpha}_1}+\nu^{-}_{\tilde{\alpha}_2}-\nu^{-}_{\tilde{\alpha}_1}\nu^{-}_{\tilde{\alpha}_2},\\&\nu^{+}_{\tilde{\alpha}_1}+\nu^{+}_{\tilde{\alpha}_2}-\nu^{+}_{\tilde{\alpha}_1}\nu^{+}_{\tilde{\alpha}_2}])\end{aligned}\tag{2}$$

$$\begin{aligned}\lambda\tilde{\alpha}_1=&([\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1];\\&[1-(1-\mu^{-}_{\tilde{\alpha}_1})^{\lambda},1-(1-\mu^{+}_{\tilde{\alpha}_1})^{\lambda}],\\&[(\nu^{-}_{\tilde{\alpha}_1})^{\lambda},(\nu^{+}_{\tilde{\alpha}_1})^{\lambda}])\end{aligned}\tag{3}$$

$$\begin{aligned}(\tilde{\alpha}_1)^{\lambda}=&([(a_1)^{\lambda},(b_1)^{\lambda},(c_1)^{\lambda},(d_1)^{\lambda}];\\&[(\mu^{-}_{\tilde{\alpha}_1})^{\lambda},(\mu^{+}_{\tilde{\alpha}_1})^{\lambda}],\\&[1-(1-\nu^{-}_{\tilde{\alpha}_1})^{\lambda},1-(1-\nu^{+}_{\tilde{\alpha}_1})^{\lambda}])\end{aligned}\tag{4}$$

### 1.3　IVITFN 的排序方法

**定义 3**[8]　对于 IVITFN，$\tilde{\alpha}=([a,b,c,d];[\mu^{-}_{\tilde{\alpha}},\mu^{+}_{\tilde{\alpha}}],[\nu^{-}_{\tilde{\alpha}},\nu^{+}_{\tilde{\alpha}}])$，其得分函数与精确函数分别为

$$S(\tilde{\alpha})=\frac{1}{8}(a+b+c+d)\cdot(\mu^{-}_{\tilde{\alpha}}-\nu^{-}_{\tilde{\alpha}}+\mu^{+}_{\tilde{\alpha}}-\nu^{+}_{\tilde{\alpha}})\tag{5}$$

$$H(\tilde{\alpha})=\frac{1}{8}(a+b+c+d)\cdot(\mu^{-}_{\tilde{\alpha}}+\nu^{-}_{\tilde{\alpha}}+\mu^{+}_{\tilde{\alpha}}+\nu^{+}_{\tilde{\alpha}})\tag{6}$$

文献[8]还给出 IVITFN 的排序方法．设 $\tilde{\alpha}_1$，$\tilde{\alpha}_2$为 2 个 IVITFN，其排序规则为

**a.** 若 $S(\tilde{\alpha}_1)>S(\tilde{\alpha}_2)$，则 $\tilde{\alpha}_1>\tilde{\alpha}_2$；

**b.** 若 $S(\tilde{\alpha}_1)=S(\tilde{\alpha}_2)$，则

(a)若 $H(\tilde{\alpha}_1)>H(\tilde{\alpha}_2)$，则 $\tilde{\alpha}_1>\tilde{\alpha}_2$；

(b)若 $H(\tilde{\alpha}_1)=H(\tilde{\alpha}_2)$，则 $\tilde{\alpha}_1=\tilde{\alpha}_2$．

### 1.4　GBM 算子

**定义 4**[15]　设 $p,q>0$，且非负实数集合$\{a_1,a_2,\cdots,a_n\}$满足

$$GB^{p,q}(a_1,a_2,\cdots,a_n)=\frac{1}{p+q}\prod_{\substack{i,j=1\\i\neq j}}^{n}(pa_i+qa_j)^{\frac{1}{n(n-1)}}\tag{7}$$

则称函数 $GB^{p,q}$为 GBM 算子．

为处理决策信息以 IVITFN 给出的 MAGMD 问题，结合 GBM 算子给出 IVITFGBM 算子．

### 1.5　IVITFGBM 算子

**定义 5**　设 $\tilde{\alpha}_i=([a_i,b_i,c_i,d_i];[\mu^{-}_{\tilde{\alpha}_i},\mu^{+}_{\tilde{\alpha}_i}],[\nu^{-}_{\tilde{\alpha}_i},\nu^{+}_{\tilde{\alpha}_i}])(i=1,2,\cdots,n)$为一组 IVITFN，且 $p,q>0$，则 IVITFGBM 算子为

$$IVITFGBM^{p,q}(\tilde{\alpha}_1,\tilde{\alpha}_2,\cdots,\tilde{\alpha}_n)=\frac{1}{p+q}\Big(\bigotimes_{\substack{i,j=1\\i\neq j}}^{n}(p\tilde{\alpha}_i\oplus q\tilde{\alpha}_j)\Big)^{\frac{1}{n(n-1)}}\tag{8}$$

**引理 1**[16]　设直觉模糊数 $\beta_{ij}=([t_{\beta_{ij}},f_{\beta_{ij}}])$，且 $i,j=1,2,\cdots,n$，$i\neq j$，则有式(9)成立．

$$\bigotimes_{\substack{i,j=1\\i\neq j}}^{n}\beta_{ij}^{\frac{1}{n(n-1)}}=\left(\prod_{\substack{i,j=1\\i\neq j}}^{n}t_{\beta_{ij}}^{\frac{1}{n(n-1)}},1-\prod_{\substack{i,j=1\\i\neq j}}^{n}(1-f_{\beta_{ij}})^{\frac{1}{n(n-1)}}\right)\tag{9}$$

式中，$t_{\beta_{ij}}$为隶属度；$f_{\beta_{ij}}$为非隶属度．

**定理 1**　设 $\tilde{\alpha}_i=([a_i,b_i,c_i,d_i];[\mu^{-}_{\tilde{\alpha}_i},\mu^{+}_{\tilde{\alpha}_i}],[\nu^{-}_{\tilde{\alpha}_i},\nu^{+}_{\tilde{\alpha}_i}])(i=1,2,\cdots,n)$为一组 IVITFN，且 $p,q>0$，则式(8)集成后的结果仍是 IVITFN，且

$$IVITFGBM^{p,q}(\tilde{\alpha}_1,\tilde{\alpha}_2,\cdots,\tilde{\alpha}_n)=([\ddot{a},\ddot{b},\ddot{c},\ddot{d}];[\ddot{\mu}^{-}_{\tilde{\alpha}},\ddot{\mu}^{+}_{\tilde{\alpha}}],[\ddot{\nu}^{-}_{\tilde{\alpha}},\ddot{\nu}^{+}_{\tilde{\alpha}}])\tag{10}$$

其中，$\ddot{a}=\frac{1}{p+q}\prod_{\substack{i,j=1\\i\neq j}}^{n}(pa_i+qa_j)^{\frac{1}{n(n-1)}}$；$\ddot{b}=\frac{1}{p+q}\prod_{\substack{i,j=1\\i\neq j}}^{n}(pb_i+qb_j)^{\frac{1}{n(n-1)}}$；$\ddot{c}=\frac{1}{p+q}\prod_{\substack{i,j=1\\i\neq j}}^{n}(pc_i+qc_j)^{\frac{1}{n(n-1)}}$；$\ddot{d}=\frac{1}{p+q}\prod_{\substack{i,j=1\\i\neq j}}^{n}(pd_i+qd_j)^{\frac{1}{n(n-1)}}$；$\ddot{\mu}^{-}_{\tilde{\alpha}}=1-\left(1-\prod_{\substack{i,j=1\\i\neq j}}^{n}(1-(1-\mu^{-}_{\tilde{\alpha}_i})^{p}(1-\mu^{-}_{\tilde{\alpha}_j})^{q})^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}$；

$\ddot{\mu}^{+}_{\tilde{\alpha}} = 1 - \left(1 - \prod_{\substack{i,j=1\\ i\neq j}}^{n} \left(1 - (1-\mu^{+}_{\tilde{\alpha}_i})^{p}(1-\mu^{+}_{\tilde{\alpha}_j})^{q}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}$；$\ddot{\nu}^{-}_{\tilde{\alpha}} = \left(1 - \prod_{\substack{i,j=1\\ i\neq j}}^{n} \left(1 - (\nu^{-}_{\tilde{\alpha}_i})^{p}(\nu^{-}_{\tilde{\alpha}_j})^{q}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}$；$\ddot{\nu}^{+}_{\tilde{\alpha}} = \left(1 - \prod_{\substack{i,j=1\\ i\neq j}}^{n} \left(1 - (\nu^{+}_{\tilde{\alpha}_i})^{p}(\nu^{+}_{\tilde{\alpha}_j})^{q}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}$.

**证明** 根据式(3)得到

$$p\tilde{\alpha}_i = ([pa_i, pb_i, pc_i, pd_i]; [1-(1-\mu^{-}_{\tilde{\alpha}_i})^{p}, 1-(1-\mu^{+}_{\tilde{\alpha}_i})^{p}], [(\nu^{-}_{\tilde{\alpha}_i})^{p}, (\nu^{+}_{\tilde{\alpha}_i})^{p}]) \tag{11}$$

$$q\tilde{\alpha}_j = ([qa_j, qb_j, qc_j, qd_j]; [1-(1-\mu^{-}_{\tilde{\alpha}_j})^{q}, 1-(1-\mu^{+}_{\tilde{\alpha}_j})^{q}], [(\nu^{-}_{\tilde{\alpha}_j})^{q}, (\nu^{+}_{\tilde{\alpha}_j})^{q}]) \tag{12}$$

进而根据式(1)得到

$$p\tilde{\alpha}_i \oplus q\tilde{\alpha}_j = ([pa_i+qa_j, pb_i+qb_j, pc_i+qc_j, pd_i+qd_j]; [1-(1-\mu^{-}_{\tilde{\alpha}_i})^{p}(1-\mu^{-}_{\tilde{\alpha}_j})^{q}, 1-(1-\mu^{+}_{\tilde{\alpha}_i})^{p}(1-\mu^{+}_{\tilde{\alpha}_j})^{q}], [(\nu^{-}_{\tilde{\alpha}_i})^{p}(\nu^{-}_{\tilde{\alpha}_j})^{q}, (\nu^{+}_{\tilde{\alpha}_i})^{p}(\nu^{+}_{\tilde{\alpha}_j})^{q}]) \tag{13}$$

$$(p\tilde{\alpha}_i \oplus q\tilde{\alpha}_j)^{\frac{1}{n(n-1)}} = \left(\left[(pa_i+qa_j)^{\frac{1}{n(n-1)}}, (pa_i+qa_j)^{\frac{1}{n(n-1)}}, (pa_i+qa_j)^{\frac{1}{n(n-1)}}, (pa_i+qa_j)^{\frac{1}{n(n-1)}}\right]; \left[(1-(1-\mu^{-}_{\tilde{\alpha}_i})^{p}(1-\mu^{-}_{\tilde{\alpha}_j})^{q})^{\frac{1}{n(n-1)}}, (1-(1-\mu^{+}_{\tilde{\alpha}_i})^{p}(1-\mu^{+}_{\tilde{\alpha}_j})^{q})^{\frac{1}{n(n-1)}}\right], \left[1-(1-(\nu^{-}_{\tilde{\alpha}_i})^{p}(\nu^{-}_{\tilde{\alpha}_j})^{q})^{\frac{1}{n(n-1)}}, 1-(1-(\nu^{+}_{\tilde{\alpha}_i})^{p}(\nu^{+}_{\tilde{\alpha}_j})^{q})^{\frac{1}{n(n-1)}}\right]\right) \tag{14}$$

根据引理1中的公式(9)，用 $p\tilde{\alpha}_i \oplus q\tilde{\alpha}_j$ 替代 $\beta_{ij}$，用 $(1-(1-\mu^{-}_{\tilde{\alpha}_i})^{p}(1-\mu^{-}_{\tilde{\alpha}_j})^{q})$，

$(1-(1-\mu^{+}_{\tilde{\alpha}_i})^{p}(1-\mu^{+}_{\tilde{\alpha}_j})^{q})$ 替代 $t_{\beta_{ij}}$，用 $(\nu^{-}_{\tilde{\alpha}_i})^{p}(\nu^{-}_{\tilde{\alpha}_j})^{q}$，$(\nu^{+}_{\tilde{\alpha}_i})^{p}(\nu^{+}_{\tilde{\alpha}_j})^{q}$ 替代 $f_{\beta_{ij}}$，结合式(1)～(4)可以证明

$$\bigotimes_{\substack{i,j=1\\ i\neq j}}^{n} (p\tilde{\alpha}_i \oplus q\tilde{\alpha}_j)^{\frac{1}{n(n-1)}} = \left(\left[\prod_{\substack{i,j=1\\ i\neq j}}^{n} (pa_i+qa_j)^{\frac{1}{n(n-1)}}, \prod_{\substack{i,j=1\\ i\neq j}}^{n} (pb_i+qb_j)^{\frac{1}{n(n-1)}}, \prod_{\substack{i,j=1\\ i\neq j}}^{n} (pc_i+qc_j)^{\frac{1}{n(n-1)}}, \prod_{\substack{i,j=1\\ i\neq j}}^{n} (pd_i+qd_j)^{\frac{1}{n(n-1)}}\right]; \left[\prod_{\substack{i,j=1\\ i\neq j}}^{n} (1-(1-\mu^{-}_{\tilde{\alpha}_i})^{p}(1-\mu^{-}_{\tilde{\alpha}_j})^{q})^{\frac{1}{n(n-1)}}, \prod_{\substack{i,j=1\\ i\neq j}}^{n} (1-(1-\mu^{+}_{\tilde{\alpha}_i})^{p}(1-\mu^{+}_{\tilde{\alpha}_j})^{q})^{\frac{1}{n(n-1)}}\right] \left[1-\prod_{\substack{i,j=1\\ i\neq j}}^{n} (1-(\nu^{-}_{\tilde{\alpha}_i})^{p}(\nu^{-}_{\tilde{\alpha}_j})^{q})^{\frac{1}{n(n-1)}}, 1-\prod_{\substack{i,j=1\\ i\neq j}}^{n} (1-(\nu^{+}_{\tilde{\alpha}_i})^{p}(\nu^{+}_{\tilde{\alpha}_j})^{q})^{\frac{1}{n(n-1)}}\right]\right) \tag{15}$$

故有

$$IVITFGBM^{p,q}(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_n) = \frac{1}{p+q}\left(\bigotimes_{\substack{i,j=1\\ i\neq j}}^{n} (p\tilde{\alpha}_i \oplus q\tilde{\alpha}_j)^{\frac{1}{n(n-1)}}\right) = ([\ddot{a}, \ddot{b}, \ddot{c}, \ddot{d}]; [\ddot{\mu}^{-}_{\tilde{\alpha}}, \ddot{\mu}^{+}_{\tilde{\alpha}}], [\ddot{\nu}^{-}_{\tilde{\alpha}}, \ddot{\nu}^{+}_{\tilde{\alpha}}]) \tag{16}$$

故式(10)得证.

容易证明 IVITFGHM 具有以下性质(限于篇幅，证明过程略)

**定理2** 幂等性

设 $IVITFN\tilde{\alpha}_i = ([a_i, b_i, c_i, d_i]; [\mu^{-}_{\tilde{\alpha}_i}, \mu^{+}_{\tilde{\alpha}_i}], [\nu^{-}_{\tilde{\alpha}_i}, \nu^{+}_{\tilde{\alpha}_i}]) = \tilde{\alpha} = ([a, b, c, d]; [\mu^{-}_{\tilde{\alpha}}, \mu^{+}_{\tilde{\alpha}}], [\nu^{-}_{\tilde{\alpha}}, \nu^{+}_{\tilde{\alpha}}])$ 对所有的 $i(i=1,2,\cdots,n)$ 有

$$IVITFGBM^{p,q}(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_n) = IVITFGBM^{p,q}(\tilde{\alpha}, \tilde{\alpha}, \cdots, \tilde{\alpha}) = \tilde{\alpha} \tag{17}$$

**定理3** 置换不变性

设 $(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_n)$ 为一组正 IVITFN 集合，$(\bar{\tilde{\alpha}}_1, \bar{\tilde{\alpha}}_2, \cdots, \bar{\tilde{\alpha}}_n)$ 为 $(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_n)$ 的任意一个置换，则有

$$IVITFGBM^{p,q}(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_n) = IVITFGBM^{p,q}(\bar{\tilde{\alpha}}_1, \bar{\tilde{\alpha}}_2, \cdots, \bar{\tilde{\alpha}}_n) \tag{18}$$

**定理4** 单调性

设

$$\tilde{\alpha}_i = ([a_i, b_i, c_i, d_i]; [\mu^{-}_{\tilde{\alpha}_i}, \mu^{+}_{\tilde{\alpha}_i}], [\nu^{-}_{\tilde{\alpha}_i}, \nu^{+}_{\tilde{\alpha}_i}])$$

和

$$\hat{\tilde{\alpha}}_i = ([\hat{a}_i, \hat{b}_i, \hat{c}_i, \hat{d}_i]; [\hat{\mu}^{-}_{\tilde{\alpha}_i}, \hat{\mu}^{+}_{\tilde{\alpha}_i}], [\hat{\nu}^{-}_{\tilde{\alpha}_i}, \hat{\nu}^{+}_{\tilde{\alpha}_i}])$$

为2个正 IVITFN 集合，如果 $a_i \geqslant \hat{a}_i, b_i \geqslant \hat{b}_i, c_i \geqslant \hat{c}_i, d_i \geqslant \hat{d}_i$，且 $\mu^{-}_{\tilde{\alpha}_i} \geqslant \hat{\mu}^{-}_{\tilde{\alpha}_i}, \mu^{+}_{\tilde{\alpha}_i} \geqslant \hat{\mu}^{+}_{\tilde{\alpha}_i}, \nu^{-}_{\tilde{\alpha}_i} \leqslant \hat{\nu}^{-}_{\tilde{\alpha}_i}, \nu^{+}_{\tilde{\alpha}_i} \leqslant \hat{\nu}^{+}_{\tilde{\alpha}_i}$，对任意的 $(i=1,2,\cdots,n)$ 都有

$$IVITFGBM^{p,q}(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_n) \geqslant IVITFGBM^{p,q}(\hat{\tilde{\alpha}}_1, \hat{\tilde{\alpha}}_2, \cdots, \hat{\tilde{\alpha}}_n) \tag{19}$$

**定理 5**　有界性

设

$\tilde{\alpha}_i=([a_i,b_i,c_i,d_i];[\mu_{\tilde{\alpha}_i}^-,\mu_{\tilde{\alpha}_i}^+],[\nu_{\tilde{\alpha}_i}^-,\nu_{\tilde{\alpha}_i}^+])$

$(i=1,2,\cdots,n)$为一正 IVITFN 集合,则有

$$\tilde{\alpha}^-\leqslant IVITFGBM^{p,q}(\tilde{\alpha}_1,\tilde{\alpha}_2,\cdots,\tilde{\alpha}_n)\leqslant\tilde{\alpha}^+ \quad (20)$$

其中,

$$\tilde{\alpha}^-=([\min_i a_i,\min_i b_i,\min_i c_i,\min_i d_i];[\min_i\mu_{\tilde{\alpha}_i}^-,\min_i\mu_{\tilde{\alpha}_i}^+],[\max_i\nu_{\tilde{\alpha}_i}^-,\max_i\nu_{\tilde{\alpha}_i}^+])\quad(i=1,2,\cdots,n)$$

$$\tilde{\alpha}^+=([\max_i a_i,\max_i b_i,\max_i c_i,\max_i d_i];[\max_i\mu_{\tilde{\alpha}_i}^-,\max_i\mu_{\tilde{\alpha}_i}^+],[\min_i\nu_{\tilde{\alpha}_i}^-,\min_i\nu_{\tilde{\alpha}_i}^+])\quad(i=1,2,\cdots,n)$$

### 1.6 IVITFGWBM 算子

基于以上研究知,IVITFGBM 算子考虑了属性间关联性,是一种 IVITFN 集成的新方法,但是在实际应用中,属性具有不同的重要程度,为此,提出 IVITFGWBM 算子.

**定义 6**　设 $\tilde{\alpha}_i=([a_i,b_i,c_i,d_i];[\mu_{\tilde{\alpha}_i}^-,\mu_{\tilde{\alpha}_i}^+],[\nu_{\tilde{\alpha}_i}^-,\nu_{\tilde{\alpha}_i}^+])(i=1,2,\cdots,n)$为一组正 IVITFN 集合,且 $p,q>0$,各属性的权重向量为$\boldsymbol{w}=(w_1,w_2,\cdots,w_n)$,且 $w_i\geqslant 0$, $\sum_{i=1}^{n}w_i=1$,则 IVITFGWBM 算子为

$$IVITFGWBM^{p,q}(\tilde{\alpha}_1,\tilde{\alpha}_2,\cdots,\tilde{\alpha}_n)=\frac{1}{p+q}\cdot\left(\bigotimes_{\substack{i,j=1\\i\neq j}}^{n}((p\tilde{\alpha}_i)^{w_i}\oplus(p\tilde{\alpha}_j)^{w_j})^{\frac{1}{n(n-1)}}\right)\quad(21)$$

**定理 6**　设 $\tilde{\alpha}_i=([a_i,b_i,c_i,d_i];[\mu_{\tilde{\alpha}_i}^-,\mu_{\tilde{\alpha}_i}^+],[\nu_{\tilde{\alpha}_i}^-,\nu_{\tilde{\alpha}_i}^+])(i=1,2,\cdots,n)$为一组正 IVITFN 集合,$p,q>0$,各属性的权重为 $\boldsymbol{w}=(w_1,w_2,\cdots,w_n)$,$w_i\geqslant 0$, $\sum_{i=1}^{n}w_i=1$,则经式(21)集成后的结果还是 IVITFN,且

$$IVITFGWBM^{p,q}(\tilde{\alpha}_1,\tilde{\alpha}_2,\cdots,\tilde{\alpha}_n)=([\dddot{a},\dddot{b},\dddot{c},\dddot{d}];[\dddot{\mu}_{\tilde{\alpha}}^-,\dddot{\mu}_{\tilde{\alpha}}^+],[\dddot{\nu}_{\tilde{\alpha}}^-,\dddot{\nu}_{\tilde{\alpha}}^+])\quad(22)$$

其中

$$\dddot{a}=\frac{1}{p+q}\prod_{\substack{i,j=1\\i\neq j}}^{n}((pa_i)^{w_i}+(qa_j)^{w_j})^{\frac{1}{n(n-1)}}$$

$$\dddot{b}=\frac{1}{p+q}\prod_{\substack{i,j=1\\i\neq j}}^{n}((pb_i)^{w_i}+(qb_j)^{w_j})^{\frac{1}{n(n-1)}}$$

$$\dddot{c}=\frac{1}{p+q}\prod_{\substack{i,j=1\\i\neq j}}^{n}((pc_i)^{w_i}+(qc_j)^{w_j})^{\frac{1}{n(n-1)}}$$

$$\dddot{d}=\frac{1}{p+q}\prod_{\substack{i,j=1\\i\neq j}}^{n}((pd_i)^{w_i}+(qd_j)^{w_j})^{\frac{1}{n(n-1)}}$$

$$\dddot{\mu}_{\tilde{\alpha}}^-=1-\left[(1-\prod_{\substack{i,j=1\\i\neq j}}^{n}(1-(1-(\mu_{\tilde{\alpha}_i}^-)^{w_i})^p\cdot(1-(\mu_{\tilde{\alpha}_j}^-)^{w_j})^q)^{\frac{1}{n(n-1)}})\right]^{\frac{1}{p+q}}$$

$$\dddot{\mu}_{\tilde{\alpha}}^+=1-\left[(1-\prod_{\substack{i,j=1\\i\neq j}}^{n}(1-(1-(\mu_{\tilde{\alpha}_i}^+)^{w_i})^p(1-(\mu_{\tilde{\alpha}_j}^+)^{w_j})^q)^{\frac{1}{n(n-1)}})\right]^{\frac{1}{p+q}}$$

$$\dddot{\nu}_{\tilde{\alpha}}^-=\left[(1-\prod_{\substack{i,j=1\\i\neq j}}^{n}(1-(1-(1-\nu_{\tilde{\alpha}_i}^-)^{w_i})^p\cdot(1-(1-\nu_{\tilde{\alpha}_j}^-)^{w_j})^q)^{\frac{1}{n(n-1)}})\right]^{\frac{1}{p+q}}$$

$$\dddot{\nu}_{\tilde{\alpha}}^+=\left[(1-\prod_{\substack{i,j=1\\i\neq j}}^{n}(1-(1-(1-\nu_{\tilde{\alpha}_i}^+)^{w_i})^p\cdot(1-(1-\nu_{\tilde{\alpha}_j}^+)^{w_j})^q)^{\frac{1}{n(n-1)}})\right]^{\frac{1}{p+q}}$$

定理 6 的证明类似定理 1,限于篇幅,证明过程略.

## 2 基于 IVITFGWBM 算子的 MAGDM 方法

基于 IVITFGWBM 算子,本文给出一种基于 IVITFN 的 MAGDM 方法,针对某 MAGDM 问题,设方案集 $A=\{A_1,A_2,\cdots,A_t\}$,属性集为 $C=\{C_1,C_2,\cdots,C_n\}$,相应属性权重为 $\boldsymbol{w}=[w_1,w_2,\cdots,w_n]^{\mathrm{T}}$,且 $w_j\in[0,1](j=1,2,\cdots,n)$. $D=\{d_1,d_2,\cdots,d_m\}$为决策集,$\boldsymbol{\omega}=[\omega_1,\omega_2,\cdots,\omega_m]^{\mathrm{T}}$ 为决策者的权重向量,$\omega_k\in[0,1](k=1,2,\cdots,m)$. 其中,$\sum_{k=1}^{m}\omega_k=1$, $\sum_{j=1}^{n}w_j=1$. 具体决策步骤如下:

**步骤 1**　设决策者 $d_k$ 给出方案$A_i$ 在属性$C_j$ 下的评价值为 IVITFN,从而得到决策矩阵为$\boldsymbol{D}^{(k)}=(\tilde{\alpha}_{ij}^{(k)})_{n\times t}$.

**步骤 2**　利用 IVITFGWBM 算子,对 $m$ 位决策专家给出的决策矩阵进行信息集成,综合专家权重,得到方案 $A_i(i=1,2,\cdots,t)$的总体评价值.

**步骤 3**　计算得分函数值与精确函数值,根据 IVITFN 排序方法对方案进行优劣排序,进而得到最佳方案.

## 3 实例分析

某食品公司拟从3个供应商($A_1,A_2,A_3$)中选择合适的合作伙伴,故针对每个供应商的4个属性进行了评估,4个属性分别为:产品质量 $C_1$;技术能力 $C_2$;污染控制 $C_3$;环境管理 $C_4$.属性的权重向量为 $\boldsymbol{w}=[0.25,0.2,0.35,0.2]^{\mathrm{T}}$,有3位决策专家 $d_1,d_2,d_3$ 分别对这 $A_1,A_2,A_3$ 根据上述的4个属性进行满意度测评,如表1~3所示.给出的测评值均为IVITFN,其中3位决策者的权重向量为 $\boldsymbol{\omega}=[0.2,0.3,0.5]^{\mathrm{T}}$.采用本文提出来的方法对供应商进行选择.

表1 决策者 $d_1$ 给出的决策矩阵

Tab.1 Decision matrix from decision maker $d_1$

| | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $C_1$ | ([0.2,0.3,0.4,0.5];[0.3,0.6],[0.1,0.3]) | ([0.1,0.3,0.4,0.5];[0.5,0.6],[0.2,0.4]) | ([0.3,0.5,0.6,0.7];[0.3,0.4],[0.2,0.3]) |
| $C_2$ | ([0.4,0.6,0.7,0.8];[0.4,0.7],[0.1,0.2]) | ([0.2,0.4,0.5,0.8];[0.4,0.5],[0.3,0.4]) | ([0.1,0.3,0.4,0.5];[0.4,0.6],[0.1,0.2]) |
| $C_3$ | ([0.3,0.5,0.6,0.8];[0.3,0.4],[0.2,0.3]) | ([0.4,0.6,0.8,1.0];[0.4,0.6],[0.2,0.4]) | ([0.2,0.3,0.4,0.5];[0.4,0.5],[0.3,0.4]) |
| $C_4$ | ([0.1,0.3,0.4,0.6];[0.6,0.7],[0.2,0.3]) | ([0.3,0.5,0.6,0.7];[0.3,0.5],[0.1,0.4]) | ([0.1,0.2,0.3,0.5];[0.2,0.5],[0.1,0.4]) |

表2 决策者 $d_2$ 给出的决策矩阵

Tab.2 Decision matrix from decision maker $d_2$

| | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $C_1$ | ([0.6,0.7,0.8,0.9];[0.6,0.7],[0.1,0.2]) | ([0.5,0.6,0.7,0.8];[0.5,0.6],[0.3,0.4]) | ([0.7,0.8,0.9,1.0];[0.5,0.8],[0.1,0.2]) |
| $C_2$ | ([0.1,0.2,0.4,0.5];[0.3,0.5],[0.2,0.4]) | ([0.4,0.5,0.6,0.8];[0.4,0.6],[0.2,0.3]) | ([0.3,0.4,0.5,0.6];[0.5,0.6],[0.2,0.4]) |
| $C_3$ | ([0.3,0.4,0.5,0.6];[0.5,0.6],[0.3,0.4]) | ([0.1,0.2,0.3,0.5];[0.4,0.6],[0.2,0.4]) | ([0.2,0.3,0.4,0.5];[0.7,0.8],[0.1,0.2]) |
| $C_4$ | ([0.1,0.2,0.4,0.5];[0.5,0.7],[0.2,0.3]) | ([0.3,0.5,0.6,0.8];[0.3,0.4],[0.1,0.2]) | ([0.2,0.3,0.4,0.5];[0.4,0.7],[0.2,0.3]) |

表3 决策者 $d_3$ 给出的决策矩阵

Tab.3 Decision matrix from decision maker $d_3$

| | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $C_1$ | ([0.4,0.5,0.6,0.7];[0.4,0.5],[0.3,0.4]) | ([0.5,0.6,0.7,0.9];[0.3,0.6],[0.1,0.2]) | ([0.3,0.4,0.5,0.6];[0.5,0.6],[0.1,0.4]) |
| $C_2$ | ([0.5,0.6,0.7,0.9];[0.2,0.4],[0.1,0.2]) | ([0.4,0.5,0.7,0.8];[0.4,0.5],[0.3,0.4]) | ([0.2,0.3,0.4,0.5];[0.4,0.5],[0.2,0.5]) |
| $C_3$ | ([0.4,0.5,0.6,0.7];[0.7,0.8],[0.1,0.2]) | ([0.2,0.3,0.4,0.5];[0.6,0.8],[0.1,0.2]) | ([0.2,0.4,0.5,0.6];[0.5,0.6],[0.2,0.4]) |
| $C_4$ | ([0.5,0.6,0.7,0.8];[0.5,0.6],[0.2,0.4]) | ([0.6,0.7,0.8,0.9];[0.3,0.4],[0.1,0.2]) | ([0.7,0.8,0.9,1.0];[0.4,0.5],[0.3,0.5]) |

**步骤1** 本文研究 $p=1,q=1$ 的情况,由式(22)算出各个决策者对每个供应商的测评值如下:

$$IVITFGWBM^{1,1}(\tilde{\alpha}_{11}^{(1)},\tilde{\alpha}_{12}^{(1)},\tilde{\alpha}_{13}^{(1)},\tilde{\alpha}_{14}^{(1)})=([0.6956,0.8026,0.8483,0.9059];[0.1156,0.1989],[0.6221,0.7274])$$

$$IVITFGWBM^{1,1}(\tilde{\alpha}_{21}^{(1)},\tilde{\alpha}_{22}^{(1)},\tilde{\alpha}_{23}^{(1)},\tilde{\alpha}_{24}^{(1)})=([0.6980,0.8194,0.8729,0.9315];[0.1203,0.1821],[0.6646,0.7976])$$

$$IVITFGWBM^{1,1}(\tilde{\alpha}_{31}^{(1)},\tilde{\alpha}_{32}^{(1)},\tilde{\alpha}_{33}^{(1)},\tilde{\alpha}_{34}^{(1)})=([0.6418,0.7509,0.8054,0.8597];[0.0946,0.1569],[0.6467,0.7564])$$

$$IVITFGWBM^{1,1}(\tilde{\alpha}_{11}^{(2)},\tilde{\alpha}_{12}^{(2)},\tilde{\alpha}_{13}^{(2)},\tilde{\alpha}_{14}^{(2)})=([0.6969,0.7710,0.8482,0.8873];[0.1517,0.2181],[0.6678,0.7547])$$

$$IVITFGWBM^{1,1}(\tilde{\alpha}_{21}^{(2)},\tilde{\alpha}_{22}^{(2)},\tilde{\alpha}_{23}^{(2)},\tilde{\alpha}_{24}^{(2)})=([0.7204,0.7940,0.8418,0.9098];[0.1203,0.1834],[0.6671,0.7583])$$

$$IVITFGWBM^{1,1}(\tilde{\alpha}_{31}^{(2)},\tilde{\alpha}_{32}^{(2)},\tilde{\alpha}_{33}^{(2)},\tilde{\alpha}_{34}^{(2)})=([0.7452,0.8029,0.8491,0.8884];[0.1756,0.2826],[0.6161,0.7174])$$

$$IVITFGWBM^{1,1}(\tilde{\alpha}_{11}^{(3)},\tilde{\alpha}_{12}^{(3)},\tilde{\alpha}_{13}^{(3)},\tilde{\alpha}_{14}^{(3)})=([0.8148,0.8573,0.8944,0.9330];[0.1476,0.2035],[0.6378,0.7342])$$

$$IVITFGWBM^{1,1}(\tilde{\alpha}_{21}^{(3)},\tilde{\alpha}_{22}^{(3)},\tilde{\alpha}_{23}^{(3)},\tilde{\alpha}_{24}^{(3)})=([0.7829,0.8323,0.8804,0.9223];[0.1235,0.2051],[0.6072,0.7009])$$

$$IVITFGWBM^{1,1}(\tilde{\alpha}_{31}^{(3)},\tilde{\alpha}_{32}^{(3)},\tilde{\alpha}_{33}^{(3)},\tilde{\alpha}_{34}^{(3)}) = ([0.737\,6,0.814\,3,0.858\,3,0.896\,0];[0.137\,2,0.182\,1],[0.662\,1,0.817\,9])$$

**步骤 2**　考虑到决策者的权重,通过加权得到　　每个供应商的测评结果如表 4 所示.

**表 4　测评结果、得分函数值和精确函数值**

**Tab.4　Evaluation results, scoring function values and precise function value**

| 方案 | 测评结果 | 得分函数值 | 精确函数值 |
|---|---|---|---|
| $A_1$ | ([0.755 6,0.820 5,0.871 3,0.913 9];[0.142 5,0.207 0],[0.643 4,0.738 9]) | −0.433 9 | 0.727 6 |
| $A_2$ | ([0.747 2,0.818 2,0.867 3,0.920 4];[0.121 9,0.194 1],[0.636 0,0.736 46]) | −0.442 8 | 0.707 7 |
| $A_3$ | ([0.720 7,0.798 2,0.845 0,0.886 5];[0.142 0,0.208 9],[0.644 9,0.774 2]) | −0.434 0 | 0.719 2 |

**步骤 3**　由式(5)、式(6)计算得分函数值和精确函数值如表 5 所示,然后根据 IVITFN 排序方法对供应商进行优劣排序为 $A_1>A_3>A_2$.因此,最优的供应商为 $A_1$.

对比分析文献[8],运用 IVITFWAA 算子和 IVITFWGA 算子的结果如表 5 所示.

**表 5　IVITFWAA 算子和 IVITFWGA 算子集成结果**

**Tab.5　Integration results of IVITFWAA operator and IVITFWGA operator**

| 方案 | IVITFWAA 算子 | | IVITFWGA 算子 | |
|---|---|---|---|---|
| | 得分函数 | 精确函数 | 得分函数 | 精确函数 |
| $A_1$ | 0.177 8 | 0.416 3 | 0.000 108 2 | 0.000 302 6 |
| $A_2$ | 0.157 1 | 0.390 8 | 0.000 107 0 | 0.000 307 4 |
| $A_3$ | 0.138 2 | 0.380 8 | 0.000 059 7 | 0.000 180 9 |

由表 5 知,通过 IVITFWAA 算子和 IVITFWGA 算子得到供应商的优劣排序均为 $A_1>A_2>A_3$.最优的供应商为 $A_1$,而 $A_2$,$A_3$ 的优劣排序与 IVITFGWBM 算子的不同,主要因为本文提出的 IVITFGWBM 算子考虑了属性间的关联性.

## 4　结束语

实际生活中的 MAGDM 问题,决策属性之间往往存在不同程度上的相互关联[17-19].针对现有 IVITF 信息集结算子存在的不足,结合 GBM 算子,研究了 IVITFGBM 算子和 IVITFGWBM 算子,并研究了 IVITFGBM 算子的一些性质,最后将 IVITFGWBM 算子应用在 MAGDM 中.算例结果表明了该算子的有效性和正确性.与传统方法对比,该方法考虑了决策属性间的关联性,使决策分析更接近决策问题的实际情况,决策结果更加合理,为解决 MAGDM 问题提供了新思路.

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