文章编号:1007－6735(2016)01－0001－07　DOI:10.13255/j.cnki.jusst.2016.01.001

一类具有时滞的反应扩散 Lotka-Volterra 竞争系统行波解的存在性

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摘要: 研究了一类具有时滞的 Lotka-Volterra 竞争系统行波解的存在性.应用具有时滞的反应扩散系统行波解存在性理论,将所研究系统行波解存在性的问题转化为寻找该系统的一对上、下解.给出了该系统在无穷远处的渐进衰减行为,完善并改进了同类系统行波解存在性的结论.

关键词: 时滞; Lotka-Volterra 竞争系统; 行波解; 存在性; 上下解
中图分类号: O 175.2　　**文献标志码:** A

Existence of Traveling Wave Solutions for Reaction-Diffusion Lotka-Volterra Competitive System with Delays

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Abstract: The existence of traveling wave solutions for two species Lotka-Volterra competitive system with delays was investigated. Based on the theory of the existence of traveling wave solutions for reaction-diffusion systems with delays, the main problem was transfered to look for a pair of upper and lower solutions for the system. And the asymptotic behavior of the system was given as an attenuated motion tending to the infinity. The study makes up and improves the results of the existence of traveling wave solutions of a class of systems.

Keywords: *delay*; *Lotka-Volterra competitive system*; *traveling wave solution*; *existence*; *upper and lower solution*

1　问题的提出

反应扩散方程在描述时空模式方面起着很重要的作用,其行波解可以用来解释自然界中的有限速度传播现象、有限震动现象等,从而备受关注[1－16].行波解的概念是 1937 年 Kolmogorov 在文献[1]中提出的,用来解释一维无穷动物栖息地上优良基因的传播过程.而具有空间扩散项的两个种群的 Lotka-Volterra 竞争系统是生物数学研究领域中比

收稿日期: 2014－08－27
基金项目: 国家自然科学基金资助项目(11071164,11101282,11271260);上海市教委科研创新项目(13ZZ118,14YZ096);上海市一流学科(系统科学)建设项目(XTKX2012);沪江基金资助项目(B14005)
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较典型且比较重要的模型之一,其模型为

$$\begin{cases}\dfrac{\partial}{\partial t}u_1(x,t)=D_1\dfrac{\partial^2}{\partial x^2}u_1(x,t)+\\ \quad r_1u_1(x,t)[1-au_1(x,t)-\\ \quad bu_2(x,t)]\\ \dfrac{\partial}{\partial t}u_2(x,t)=D_2\dfrac{\partial^2}{\partial x^2}u_2(x,t)+\\ \quad r_2u_2(x,t)[1-a_0u_2(x,t)-\\ \quad b_0u_1(x,t)]\end{cases}\tag{1}$$

式中:$u_1(x,t)$,$u_2(x,t)$分别表示两个竞争物种在时间t、位置x处的物种密度;D_1,D_2分别表示两个物种的空间扩散率;r_1,r_2分别表示两个物种的固有增长率;a,a_0分别表示物种内部的拥挤系数;b,b_0分别表示两个物种之间的竞争系数;系统(1)的中所有系数均为正常数.

许多学者对系统(1)的行波解作了广泛的研究[2-8],文献[2,5,8]分别利用相平面分析法、指数同伦法、度理论的技巧得到了系统(1)的行波解的存在性.

物种的繁殖由于受到妊娠、环境及成熟过程等各方面因素的影响,物种密度在时间上的滞后是在所难免的,所以,具有时滞的Lotka-Volterra竞争系统得到了学者们的关注.

$$\begin{cases}\dfrac{\partial}{\partial t}u_1(x,t)=D_1\dfrac{\partial^2}{\partial x^2}u_1(x,t)+\\ \quad r_1u_1(x,t)[1-au_1(x,t-\tau_1)-\\ \quad bu_2(x,t-\tau_2)]\\ \dfrac{\partial}{\partial t}u_2(x,t)=D_2\dfrac{\partial^2}{\partial x^2}u_2(x,t)+\\ \quad r_2u_2(x,t)[1-a_0u_2(x,t-\tau_3)-\\ \quad b_0u_1(x,t-\tau_4)]\end{cases}\tag{2}$$

式中,τ_1,τ_2,τ_3,τ_4表示反馈时滞.

但是,先前的一些方法不能够运用到类似系统(2)这样具有时滞的反应扩散Lotka-Volterra竞争系统中去.于是,Lü等[9]利用单调迭代方法和上下解的技巧研究了系统(2)的行波解;Li等[10]利用Shauder不动点定理和新的交叉迭代的方法得出了系统(2)的行波解的存在性.

在自然界中,两个物种的种群数量受到各自的固有增长率的影响,与此同时,在时间t,$t-\tau_i(i=1,2,3,4)$自身的以及竞争物种的种群密度增加时,两个物种的种群密度增长率都会下降,这表明两个竞争物种的种群密度均分别受到自身在时间t,$t-\tau_i(i=1,3)$以及竞争物种在时间t,$t-\tau_i(i=2,4)$时种群密度的影响.受到学者们对式(1)和式(2)的研究的引导,以及文献[10-11]的结论的启发,本文研究以下的Lotka-Volterra竞争系统:

$$\begin{cases}\dfrac{\partial}{\partial t}u_1(x,t)=D_1\dfrac{\partial^2}{\partial x^2}u_1(x,t)+r_1u_1(x,t)\cdot\\ \quad [1-a_1u_1(x,t)-b_1u_1(x,t-\tau_1)-\\ \quad c_1u_2(x,t)-d_1u_2(x,t-\tau_2)]\\ \dfrac{\partial}{\partial t}u_2(x,t)=D_2\dfrac{\partial^2}{\partial x^2}u_2(x,t)+r_2u_2(x,t)\cdot\\ \quad [1-a_2u_2(x,t)-b_2u_2(x,t-\tau_3)-\\ \quad c_2u_1(x,t)-d_2u_1(x,t-\tau_4)]\end{cases}\tag{3}$$

式中:a_1,b_1,a_2,b_2表示物种内部的拥挤系数;c_1,d_1,c_2,d_2表示两个物种之间的竞争系数.

综合考虑现实物种的生长受到多方面内部及外部因素的影响,系统(3)这个模型具有更强的现实生物意义.

2 预备知识

具有时滞的反应扩散系统的一般形式为

$$\begin{cases}\dfrac{\partial}{\partial t}u_1(x,t)=D_1\dfrac{\partial^2}{\partial x^2}u_1(x,t)+\\ \quad f_1(u_{1t}(x),u_{2t}(x))\\ \dfrac{\partial}{\partial t}u_2(x,t)=D_2\dfrac{\partial^2}{\partial x^2}u_2(x,t)+\\ \quad f_2(u_{1t}(x),u_{2t}(x))\end{cases}\tag{4}$$

式中:$D_1,D_2>0$;$f_i:\mathbb{R}^2\to\mathbb{R}$,$i=1,2$,且$f_i$是连续函数.

系统(4)的行波解是一对形如$u_1(x,t)=\varphi(x+ct)$,$u_2(x,t)=\psi(x+ct)$的特解,其中,$(\varphi,\psi)\in C^2(\mathbb{R},\mathbb{R}^2)$,$c>0$,$c$是波速.将$u_1(x,t)=\varphi(x+ct)$,$u_2(x,t)=\psi(x+ct)$代入系统(4)且仍然用$t$来代替$x+ct$,则系统(4)的行波解满足下面的泛函微分方程:

$$\begin{cases}c\varphi'(t)=D_1\varphi''(t)+f_1^c(\varphi_t,\psi_t)\\ c\psi'(t)=D_2\psi''(t)+f_2^c(\varphi_t,\psi_t)\end{cases}\tag{5}$$

其中,$f_i^c:C([-\tau,0],\mathbb{R}^2)\to\mathbb{R}$,$i=1,2$,定义为

$$f_i^c(\varphi_t(s),\psi_t(s))=f_i(\varphi_t^c(s),\psi_t^c(s))=\\ f_i(\varphi(t+cs),\psi(t+cs)),s\in[-\tau,0]$$

设$E_0=(0,0)$,$E^*=(k_1,k_2)$是系统(5)的2个平衡点,其中,$k_1,k_2>0$.所以,系统(4)有行波解当且仅当系统(5)有满足边值条件

$$\begin{cases}\lim\limits_{t\to-\infty}(\varphi(t),\psi(t))=(0,0)\\ \lim\limits_{t\to+\infty}(\varphi(t),\psi(t))=(k_1,k_2)\end{cases}\tag{6}$$

的解.

现给出一些假设：

A1　$f_i(0,0)=f_i(k_1,k_2)=0, i=1,2.$

A2　存在 2 个常数 $L_1, L_2>0$，使得

$$|f_1(\varphi_1,\psi_1)-f_1(\varphi_2,\psi_2)|\leqslant L_1\|\Phi-\Psi\|$$
$$|f_2(\varphi_1,\psi_1)-f_2(\varphi_2,\psi_2)|\leqslant L_2\|\Phi-\Psi\|$$

其中，$\Phi=(\varphi_1,\psi_1)$，$\Psi=(\varphi_2,\psi_2)$，$\Phi,\Psi\in C([-\tau,0],\mathbb{R}^2)$，$0\leqslant\varphi_i(s),\psi_i(s)\leqslant M_i$，$s\in[-\tau,0]$，$M_i>k_i>0, i=1,2.$

A3　(文献[10,12]中的 WQM* 条件)，存在 $\beta_1, \beta_2>0$，使得

$$f_1(\varphi_1(s),\psi_1(s))-f_1(\varphi_2(s),\psi_1(s))+\beta_1[\varphi_1(0)-\varphi_2(0)]\geqslant 0$$
$$f_1(\varphi_1(s),\psi_1(s))-f_1(\varphi_1(s),\psi_2(s))\leqslant 0$$
$$f_2(\varphi_1(s),\psi_1(s))-f_2(\varphi_1(s),\psi_2(s))+\beta_2[\psi_1(0)-\psi_2(0)]\geqslant 0$$
$$f_2(\varphi_1(s),\psi_1(s))-f_2(\varphi_1(s),\psi_2(s))\leqslant 0$$

其中，$\varphi_1(s),\psi_1(s),\varphi_2(s),\psi_2(s)\in C([-\tau,0],\mathbb{R})$，且满足

a.　$0\leqslant\varphi_2(s)\leqslant\varphi_1(s)\leqslant M_1$，$0\leqslant\psi_2(s)\leqslant\psi_1(s)\leqslant M_2$，$s\in[-\tau,0]$；

b.　$e^{\beta_1 s}[\varphi_1(s)-\varphi_2(s)]$，$e^{\beta_2 s}[\psi_1(s)-\psi_2(s)]$在 $s\in[-\tau,0]$上都是不减的.

为了方便起见，记 $C_{[\boldsymbol{O},\boldsymbol{M}]}(\mathbb{R},\mathbb{R}^2)=\{(\varphi,\psi)\in C(\mathbb{R},\mathbb{R}^2): 0\leqslant\varphi(s)\leqslant M_1, 0\leqslant\psi(s)\leqslant M_2, s\in\mathbb{R}\}$. 其中，$\boldsymbol{O}=(0,0)$，$\boldsymbol{M}=(M_1,M_2)$.

定义集合

$$\Gamma^*([\underline{\varphi},\underline{\psi}],[\bar{\varphi},\bar{\psi}])=\left\{(\varphi,\psi)\in C_{[\boldsymbol{O},\boldsymbol{M}]}(\mathbb{R},\mathbb{R}^2)\left|\begin{array}{l}(\text{i})\ \underline{\varphi}(t)\leqslant\varphi(t)\leqslant\bar{\varphi}(t),\underline{\psi}(t)\leqslant\psi(t)\leqslant\bar{\psi}(t),t\in\mathbb{R}.\\(\text{ii})\ e^{\beta_1 t}[\varphi(t)-\underline{\varphi}(t)],e^{\beta_1 t}[\bar{\varphi}(t)-\varphi(t)],\\ \qquad e^{\beta_2 t}[\psi(t)-\underline{\psi}(t)],e^{\beta_2 t}[\bar{\psi}(t)-\psi(t)],\\ \qquad \text{均是不减的}, t\in\mathbb{R}.\end{array}\right.\right\}$$

假设系统(5)有一对上、下解 $\bar{\Phi}=(\bar{\varphi},\bar{\psi})$，$\underline{\Phi}=(\underline{\varphi},\underline{\psi})$，$\bar{\Phi},\underline{\Phi}\in C(\mathbb{R},\mathbb{R}^2)$且满足

P1　$(0,0)\leqslant(\underline{\varphi}(t),\underline{\psi}(t))\leqslant(\bar{\varphi}(t),\bar{\psi}(t))\leqslant(M_1,M_2)$

P2　$\lim\limits_{t\to-\infty}(\bar{\varphi}(t),\bar{\psi}(t))=(0,0)$

$\lim\limits_{t\to+\infty}(\underline{\varphi}(t),\underline{\psi}(t))=\lim\limits_{t\to+\infty}(\bar{\varphi}(t),\bar{\psi}(t))=(k_1,k_2)$

P3　$e^{\beta_1 t}[\bar{\varphi}(t)-\underline{\varphi}(t)]$，$e^{\beta_2 t}[\bar{\psi}(t)-\underline{\psi}(t)]$，均是不减的，$t\in\mathbb{R}$.

定义 1[10,12]　若存在常数 $T_i, i=1,2,\cdots,m$，使得 $\bar{\Phi}=(\bar{\varphi},\bar{\psi})$，$\underline{\Phi}=(\underline{\varphi},\underline{\psi})$在 $\mathbb{R}\backslash\{T_i: i=1,2,\cdots,m\}$上是二次连续可微函数，且满足

$$\begin{cases}D_1\bar{\varphi}''(t)-c\bar{\varphi}'(t)+f_1^c(\bar{\varphi}_t,\underline{\psi}_t)\leqslant 0,\\ \qquad t\in\mathbb{R}\backslash\{T_i: i=1,2,\cdots,m\}\\ D_2\bar{\psi}''(t)-c\bar{\psi}'(t)+f_2^c(\underline{\varphi}_t,\bar{\psi}_t)\leqslant 0,\\ \qquad t\in\mathbb{R}\backslash\{T_i: i=1,2,\cdots,m\}\end{cases}\tag{7}$$

$$\begin{cases}D_1\underline{\varphi}''(t)-c\underline{\varphi}'(t)+f_1^c(\underline{\varphi}_t,\bar{\psi}_t)\geqslant 0,\\ \qquad t\in\mathbb{R}\backslash\{T_i: i=1,2,\cdots,m\}\\ D_2\underline{\psi}''(t)-c\underline{\psi}'(t)+f_2^c(\bar{\varphi}_t,\underline{\psi}_t)\geqslant 0,\\ \qquad t\in\mathbb{R}\backslash\{T_i: i=1,2,\cdots,m\}\end{cases}\tag{8}$$

则称 $\bar{\Phi}=(\bar{\varphi},\bar{\psi})$、$\underline{\Phi}=(\underline{\varphi},\underline{\psi})$ 为系统(5)的一对弱上、下解.

定理 1[10,12]　如果系统(5)满足 A1，A2，WQM* 条件，有一对弱的上、下解 $\bar{\Phi}=(\bar{\varphi},\bar{\psi})$，$\underline{\Phi}=(\underline{\varphi},\underline{\psi})$，$\bar{\Phi},\underline{\Phi}\in C(\mathbb{R},\mathbb{R}^2)$且满足 P1－P3，并满足下式：

$$\begin{cases}\bar{\varphi}'(t+)\leqslant\bar{\varphi}'(t-),\bar{\psi}'(t+)\leqslant\bar{\psi}'(t-)\\ \underline{\varphi}'(t+)\geqslant\underline{\varphi}'(t-),\underline{\psi}'(t+)\geqslant\underline{\psi}'(t-)\end{cases}\tag{9}$$

则对任意的 $c>1-\min\{\beta_1 d_1,\beta_2 d_2\}$，系统(5)有满足边值条件(6)的行波解.

文献[13－14]进一步将具有时滞的反应扩散系统(4)的行波解的存在性推广到具有时滞的 n 个方程组和时空时滞的方程组的行波解的存在性.

3　系统(3)的行波解的存在性

现运用定理 1 来确定系统(3)的行波解的存在性. 假设 $a_1+b_1>c_2+d_2$，$a_2+b_2>c_1+d_1$，那么，系统(3)有 4 个平衡点：$\boldsymbol{E}_1=(0,0)$，$\boldsymbol{E}_2=\left(0,\frac{1}{a_2+b_2}\right)$，$\boldsymbol{E}_3=\left(\frac{1}{a_1+b_1},0\right)$，$\boldsymbol{E}_4=(k_1,k_2)$，其中

$$k_1=\frac{(a_2+b_2)-(c_1+d_1)}{(a_1+b_1)(a_2+b_2)-(c_1+d_1)(c_2+d_2)}>0$$

$$k_2=\frac{(a_1+b_1)-(c_2+d_2)}{(a_1+b_1)(a_2+b_2)-(c_1+d_1)(c_2+d_2)}>0$$

将 $u_1(x,t)=\varphi(x+ct)$，$u_2(x,t)=\psi(x+ct)$ 代入系统(3)且仍然用 t 来代替 $x+ct$，得

$$\begin{cases} c\varphi'(t)=D_1\varphi''(t)+r_1\varphi(t)[1-a_1\varphi(t)- \\ \quad b_1\varphi(t-c\tau_1)-c_1\psi(t)-d_1\psi(t-c\tau_2)] \\ c\psi'(t)=D_2\psi''(t)+r_2\psi(t)[1-a_2\psi(t)- \\ \quad b_2\psi(t-c\tau_3)-c_2\varphi(t)-d_2\varphi(t-c\tau_4)] \end{cases} \tag{10}$$

本文将研究系统(3)连接平衡点 E_1，E_4 的行波解的存在性，即研究系统(10)满足边值条件

$$\begin{cases} \lim\limits_{t\to-\infty}\varphi(t)=0, & \lim\limits_{t\to+\infty}\varphi(t)=k_1 \\ \lim\limits_{t\to-\infty}\psi(t)=0, & \lim\limits_{t\to+\infty}\psi(t)=k_2 \end{cases} \tag{11}$$

的解的存在性.

对 $\varphi,\psi\in([-\tau,0],\mathbb{R})$，其中，$\tau=\max\{\tau_1,\tau_2,\tau_3,\tau_4\}$，记

$$\begin{cases} f_1(\varphi,\psi)=r_1\varphi(0)[1-a_1\varphi(0)- \\ \quad b_1\varphi(-\tau_1)-c_1\psi(0)-d_1\psi(-\tau_2)] \\ f_2(\varphi,\psi)=r_2\psi(0)[1-a_2\psi(0)- \\ \quad b_2\psi(-\tau_3)-c_2\varphi(0)-d_2\varphi(-\tau_4)] \end{cases} \tag{12}$$

易得 f_1，f_2 满足假设 A1 和 A2.

3.1 系统(10)满足 WQM* 条件

引理 1 当 $\tau_1,\tau_3>0$ 且充分小时，系统(12)中的函数 (f_1,f_2) 满足 WQM* 条件.

证明 令 $\Phi(s)=(\varphi_1(s),\varphi_2(s))$，$\Psi(s)=(\psi_1(s),\psi_2(s))$，$\Phi(s),\Psi(s)\in C([-\tau,0],\mathbb{R}^2)$，且

a. $0\leqslant\varphi_2(s)\leqslant\varphi_1(s)\leqslant M_1$，$0\leqslant\psi_2(s)\leqslant\psi_1(s)\leqslant M_2$，$s\in[-\tau,0]$；

b. $e^{\beta_1 s}[\varphi_1(s)-\varphi_2(s)]$，$e^{\beta_2 s}[\psi_1(s)-\psi_2(s)]$ 不减，$s\in[-\tau,0]$.

于是

$$\begin{aligned} &f_1(\varphi_1,\psi_1)-f_1(\varphi_2,\psi_1)=r_1\varphi_1(0)[1- \\ &\quad a_1\varphi_1(0)-b_1\varphi_1(-\tau_1)-c_1\psi_1(0)- \\ &\quad d_1\psi_1(-\tau_2)]-r_1\varphi_2(0)[1-a_1\varphi_2(0)- \\ &\quad b_1\varphi_2(-\tau_1)-c_1\psi_1(0)-d_1\psi_1(-\tau_2)]= \\ &\quad r_1[\varphi_1(0)-\varphi_2(0)]-r_1d_1\psi_1(-\tau_2)\cdot \\ &\quad [\varphi_1(0)-\varphi_2(0)]-r_1a_1[\varphi_1^2(0)-\varphi_2^2(0)]- \\ &\quad r_1b_1[\varphi_1(0)\varphi_1(-\tau_1)-\varphi_2(0)\varphi_2(-\tau_1)]- \\ &\quad r_1c_1\psi_1(0)[\varphi_1(0)-\varphi_2(0)]\geqslant(r_1- \\ &\quad r_1d_1M_2-2r_1a_1M_1-r_1c_1M_2)[\varphi_1(0)- \\ &\quad \varphi_2(0)]-r_1b_1\varphi_2(0)[\varphi_1(-\tau_1)- \\ &\quad \varphi_2(-\tau_1)]-r_1b_1\varphi_1(-\tau_1)[\varphi_1(0)-\varphi_2(0)]\geqslant \\ &\quad r_1(1-2a_1M_1-b_1M_1-c_1M_2-d_1M_2)\cdot \\ &\quad [\varphi_1(0)-\varphi_2(0)]-r_1b_1\varphi_2(0)e^{\beta_1\tau_1}e^{-\beta_1\tau_1}\cdot \\ &\quad [\varphi_1(-\tau_1)-\varphi_2(-\tau_1)]\geqslant r_1(1-2a_1M_1- \\ &\quad b_1M_1-c_1M_2-d_1M_2-b_1M_1\cdot \\ &\quad e^{\beta_1\tau_1})[\varphi_1(0)-\varphi_2(0)] \end{aligned}$$

当 $\tau_1>0$ 且充分小，则存在 $\beta_1>0$，使得

$$r_1(1-2a_1M_1-b_1M_1-c_1M_2-d_1M_2-b_1M_1e^{\beta_1\tau_1})>-\beta_1$$

因此

$$f_1(\varphi_1,\psi_1)-f_1(\varphi_2,\psi_1)+\beta_1[\varphi_1(0)-\varphi_2(0)]\geqslant 0$$

同理可得，当 $\tau_3>0$ 且充分小，则存在 $\beta_2>0$，使得

$$f_2(\varphi_1,\psi_1)-f_2(\varphi_1,\psi_2)+\beta_2[\psi_1(0)-\psi_2(0)]\geqslant 0$$

另外

$$\begin{aligned} &f_1(\varphi_1,\psi_1)-f_1(\varphi_1,\psi_2)=r_1\varphi_1(0)\cdot \\ &\quad [1-a_1\varphi_1(0)-b_1\varphi_1(-\tau_1)-c_1\psi_1(0)- \\ &\quad d_1\psi_1(-\tau_2)]-r_1\varphi_1(0)[1-a_1\varphi_1(0)- \\ &\quad b_1\varphi_1(-\tau_1)-c_1\psi_2(0)-d_1\psi_2(-\tau_2)]= \\ &\quad -r_1c_1\varphi_1(0)[\psi_1(0)-\psi_2(0)]-r_1d_1\varphi_1(0)\cdot \\ &\quad [\psi_1(-\tau_2)-\psi_2(-\tau_2)]\leqslant 0 \end{aligned}$$

同理可得，$f_2(\varphi_1,\psi_1)-f_2(\varphi_1,\psi_2)\leqslant 0$.

引理 1 得证.

3.2 系统(10)的一对弱上、下解

设 λ_1,λ_2 是方程 $D_1\lambda^2-c\lambda+r_1=0$ 的 2 个根，且 $0<\lambda_1<\lambda_2$，设 λ_3,λ_4 是方程 $D_2\lambda^2-c\lambda+r_2=0$ 的 2 个根，且 $0<\lambda_3<\lambda_4$，其中，$c>2\max\{\sqrt{r_1D_1},\sqrt{r_2D_2}\}$. 令

$$\eta\in\left(1,\min\left\{2,\frac{\lambda_2}{\lambda_1},\frac{\lambda_4}{\lambda_3},\frac{\lambda_1+\lambda_3}{\lambda_1},\frac{\lambda_1+\lambda_3}{\lambda_3}\right\}\right)$$

$$\begin{cases} l_1(t)=e^{\lambda_1 t}-q\,e^{\eta\lambda_1 t} \\ l_3(t)=e^{\lambda_3 t}-q\,e^{\eta\lambda_3 t} \end{cases}$$

其中，q 是充分大的正常数，易见，$l_1(t)$，$l_3(t)$ 均有全局最大值，分别记为 m_1，$m_2>0$，令 $t_i=\max\{t:l_i(t)=m_i\}$，$i=1,3$，则 $t_i=\dfrac{1}{\eta\lambda_i-\lambda_i}\ln\dfrac{1}{q\eta}<0$，$q$ 充分大，$i=1,3$. 易得，$l_i(t)$ 在 $(0,t_i)$ 上单调递增，在 $(t_i,+\infty)$ 上单调递减，$i=1,3$. 则对任意给定的 λ，存在 $\varepsilon_2>0$，$\varepsilon_4>0$，使得

$$\begin{cases} k_1-\varepsilon_2e^{-\lambda t_1}=l_1(t_1)=m_1 \\ k_2-\varepsilon_4e^{-\lambda t_3}=l_3(t_3)=m_3 \end{cases}$$

又因为 $a_1+b_1>c_2+d_2$，$a_2+b_2>c_1+d_1$，则存在 $\varepsilon_0,\varepsilon_1,\varepsilon_3>0$，使得

$$\begin{cases} (a_1+b_1)\varepsilon_1-(c_1+d_1)\varepsilon_4>\varepsilon_0 \\ (a_1+b_1)\varepsilon_2-(c_1+d_1)\varepsilon_3>\varepsilon_0 \\ (a_2+b_2)\varepsilon_3-(c_1+d_1)\varepsilon_2>\varepsilon_0 \\ (a_2+b_2)\varepsilon_4-(c_2+d_2)\varepsilon_1>\varepsilon_0 \end{cases} \tag{13}$$

对以上的常数以及适当的常数 $t_2, t_4>0$,定义以下连续函数:

$$\begin{cases}\overline{\varphi}(t)=\begin{cases}e^{\lambda_1 t}, & t\leqslant t_2\\ k_1+\varepsilon_1 e^{-\lambda t}, & t\geqslant t_2\end{cases}\\ \overline{\psi}(t)=\begin{cases}e^{\lambda_3 t}, & t\leqslant t_4\\ k_2+\varepsilon_3 e^{-\lambda t}, & t\geqslant t_4\end{cases}\end{cases}\tag{14}$$

$$\begin{cases}\underline{\varphi}(t)=\begin{cases}e^{\lambda_1 t}-q\,e^{\eta\lambda_1 t}, & t\leqslant t_1\\ k_1-\varepsilon_2 e^{-\lambda t}, & t\geqslant t_1\end{cases}\\ \underline{\psi}(t)=\begin{cases}e^{\lambda_3 t}-q\,e^{\eta\lambda_3 t}, & t\leqslant t_3\\ k_2-\varepsilon_4 e^{-\lambda t}, & t\geqslant t_3\end{cases}\end{cases}\tag{15}$$

其中,$q>0$ 且是充分大的数,$\lambda>0$ 且是充分小的数.易得 $M_1=\sup\limits_{t\in\mathbb{R}}\overline{\varphi}(t)>k_1, M_2=\sup\limits_{t\in\mathbb{R}}\overline{\psi}(t)>k_2$.且 $\overline{\varphi}(t),\underline{\varphi}(t),\overline{\psi}(t),\underline{\psi}(t)$ 满足 P1 - P3 以及式(9).对足够大的 $q>0$ 和充分小的 $\lambda>0$,有

$$\min\{t_2,t_4\}-\max\{c\tau_1,c\tau_2,c\tau_3,c\tau_4\}\geqslant\max\{t_1,t_3\}$$

引理 2　假设式(13)成立,当 $\tau_1,\tau_3>0$ 充分小时,$\overline{\Phi}(t)=(\overline{\varphi}(t),\overline{\psi}(t))$, $\underline{\Phi}(t)=(\underline{\varphi}(t),\underline{\psi}(t))$是系统(10)的一对弱上、下解.

证明　首先证明 $\overline{\Phi}(t)=(\overline{\varphi}(t),\overline{\psi}(t))$是系统(10)的一个弱上解.

a. 当 $t\leqslant t_2$时,$\overline{\varphi}(t)=e^{\lambda_1 t}$,

$$\begin{aligned}&D_1\overline{\varphi}''(t)-c\overline{\varphi}'(t)+r_1\overline{\varphi}(t)[1-a_1\overline{\varphi}(t)-\\&\quad b_1\overline{\varphi}(t-c\tau_1)-c_1\underline{\psi}(t)-d_1\underline{\psi}(t-c\tau_2)]\leqslant\\&\quad D_1\overline{\varphi}''(t)-c\overline{\varphi}'(t)+r_1\overline{\varphi}(t)=\\&\quad[D_1\lambda_1^2-c\lambda_1+r_1]e^{\lambda_1 t}=0\end{aligned}$$

b. 当 $t\geqslant t_2+c\tau_1$时,有 $t-c\tau_2\geqslant t_3$,

$$\begin{aligned}&\overline{\varphi}t)=k_1+\varepsilon_1 e^{-\lambda t}\\&\overline{\varphi}(t-c\tau_1)=k_1+\varepsilon_1 e^{-\lambda(t-c\tau_1)}\\&\underline{\psi}(t)=k_2-\varepsilon_4 e^{-\lambda t}\\&\underline{\psi}(t-c\tau_2)=k_2-\varepsilon_4 e^{-\lambda(t-c\tau_2)}\end{aligned}$$

$$\begin{aligned}&D_1\overline{\varphi}''(t)-c\overline{\varphi}'(t)+r_1\overline{\varphi}(t)[1-a_1\overline{\varphi}(t)-\\&\quad b_1\overline{\varphi}(t-c\tau_1)-c_1\underline{\psi}(t)-d_1\underline{\psi}(t-c\tau_2)]=\\&\quad D_1(k_1+\varepsilon_1 e^{-\lambda t})''-c(k_1+\varepsilon_1 e^{-\lambda t})'+\\&\quad r_1(k_1+\varepsilon_1 e^{-\lambda t})[1-a_1(k_1+\varepsilon_1 e^{-\lambda t})-\\&\quad b_1(k_1+\varepsilon_1 e^{-\lambda(t-c\tau_1)})-c_1(k_2-\varepsilon_4 e^{-\lambda t})-\\&\quad d_1(k_2-\varepsilon_4 e^{-\lambda(t-c\tau_2)}))]=e^{-\lambda t}(D_1\varepsilon_1\lambda^2+c\varepsilon_1\lambda)+\\&\quad r_1(k_1+\varepsilon_1 e^{-\lambda t})[1-a_1(k_1+\varepsilon_1 e^{-\lambda t})-\\&\quad b_1(k_1+\varepsilon_1 e^{-\lambda(t-c\tau_1)})-c_1(k_2-\varepsilon_4 e^{-\lambda t})-\\&\quad d_1(k_2-\varepsilon_4 e^{-\lambda(t-c\tau_2)})]=I_1(\lambda)\end{aligned}$$

显然,$I_1(0)=r_1(k_1+\varepsilon_1)[1-(a_1+b_1)k_1-(c_1+d_1)k_2-(a_1+b_1)\varepsilon_1-(c_1+d_1)\varepsilon_4]=r_1(k_1+\varepsilon_1)[-(a_1+b_1)\varepsilon_1-(c_1+d_1)\varepsilon_4]$.

又因为 $(a_1+b_1)\varepsilon_1-(c_1+d_1)\varepsilon_4>\varepsilon_0$,则 $I_1(0)<0$,则存在 $\overline{\lambda}_1^*>0$,使得 $I_1(\lambda)<0,\lambda\in(0,\overline{\lambda}_1^*)$,即

$$\begin{aligned}&D_1\overline{\varphi}''(t)-c\overline{\varphi}'(t)+r_1\overline{\varphi}(t)[1-a_1\overline{\varphi}(t)-\\&\quad b_1\overline{\varphi}(t-c\tau_1)-c_1\underline{\psi}(t)-\\&\quad d_1\underline{\psi}(t-c\tau_2)]\leqslant 0,\lambda\in(0,\overline{\lambda}_1^*)\end{aligned}$$

c. 当 $t_2<t<t_2+c\tau_1$时,有 $t-c\tau_2\geqslant t_3$,

$$\begin{aligned}&\overline{\varphi}(t)=k_1+\varepsilon_1 e^{-\lambda t}\\&\overline{\varphi}(t-c\tau_1)=e^{\lambda_1(t-c\tau_1)}\\&\underline{\psi}(t)=k_2-\varepsilon_4 e^{-\lambda t}\\&\underline{\psi}(t-c\tau_2)=k_2-\varepsilon_4 e^{-\lambda(t-c\tau_2)}\end{aligned}$$

$$\begin{aligned}&D_1\overline{\varphi}''(t)-c\overline{\varphi}'(t)+r_1\overline{\varphi}(t)[1-a_1\overline{\varphi}(t)-\\&\quad b_1\overline{\varphi}(t-c\tau_1)-c_1\underline{\psi}(t)-d_1\underline{\psi}(t-c\tau_2)]=\\&\quad D_1(k_1+\varepsilon_1 e^{-\lambda t})''-c(k_1+\varepsilon_1 e^{-\lambda t})'+\\&\quad r_1(k_1+\varepsilon_1 e^{-\lambda t})[1-a_1(k_1+\varepsilon_1 e^{-\lambda t})-\\&\quad b_1 e^{\lambda_1(t-c\tau_1)}-c_1(k_2-\varepsilon_4 e^{-\lambda t})-\\&\quad d_1(k_2-\varepsilon_4 e^{-\lambda(t-c\tau_2)})]=e^{-\lambda t}\cdot\\&\quad(D_1\varepsilon_1\lambda^2+c\varepsilon_1\lambda)+r_1(k_1+\varepsilon_1 e^{-\lambda t})\cdot\\&\quad[(1-a_1k_1-c_1k_2-d_1k_2)-a_1\varepsilon_1 e^{-\lambda t}-\\&\quad b_1 e^{\lambda_1(t-c\tau_1)}+c_1\varepsilon_4 e^{-\lambda t}+d_1\varepsilon_4 e^{-\lambda(t-c\tau_2)}]=\\&\quad e^{-\lambda t}(D_1\varepsilon_1\lambda^2+c\varepsilon_1\lambda)+r_1(k_1+\varepsilon_1 e^{-\lambda t})\cdot\\&\quad[b_1k_1-a_1\varepsilon_1 e^{-\lambda t}-b_1 e^{\lambda_1(t-c\tau_1)}+\\&\quad c_1\varepsilon_4 e^{-\lambda t}+d_1\varepsilon_4 e^{-\lambda(t-c\tau_2)}]\leqslant\\&\quad e^{-\lambda t}(D_1\varepsilon_1\lambda^2+c\varepsilon_1\lambda)+r_1(k_1+\varepsilon_1 e^{-\lambda t})\cdot\\&\quad[b_1k_1-a_1\varepsilon_1 e^{-\lambda t}-b_1 e^{\lambda_1(t_2-c\tau_1)}+\\&\quad c_1\varepsilon_4 e^{-\lambda t}+d_1\varepsilon_4 e^{-\lambda(t-c\tau_2)}]=\\&\quad e^{-\lambda t}(D_1\varepsilon_1\lambda^2+c\varepsilon_1\lambda)+r_1(k_1+\varepsilon_1 e^{-\lambda t})\cdot\\&\quad[b_1k_1-a_1\varepsilon_1 e^{-\lambda t}-b_1 e^{-\lambda_1 c\tau_1}(k_1+\varepsilon_1 e^{-\lambda t_2})+\\&\quad c_1\varepsilon_4 e^{-\lambda t}+d_1\varepsilon_4 e^{-\lambda(t-c\tau_2)}]=I_2(\lambda)\end{aligned}$$

于是

$$\begin{aligned}I_2(0)=&\,r_1(k_1+\varepsilon_1)[b_1k_1-a_1\varepsilon_1-\\&\,b_1 e^{-\lambda_1 c\tau_1}(k_1+\varepsilon_1)+(c_1+d_1)\varepsilon_4]\end{aligned}$$

对于充分小的 $\tau_1>0$,存在 $0<\varepsilon^*<\dfrac{\varepsilon_0}{b_1(k_1+\varepsilon_1)}<1$,使得 $e^{-\lambda_1 c\tau_1}>1-\varepsilon^*>0$,因此,

$$\begin{aligned}I_2(0)\leqslant&\,r_1(k_1+\varepsilon_1)[b_1k_1-a_1\varepsilon_1-\\&\,b_1(1-\varepsilon^*)(k_1+\varepsilon_1)+(c_1+d_1)\varepsilon_4]=\\&\,r_1(k_1+\varepsilon_1)[-(a_1+b_1)\varepsilon_1+\\&\,(c_1+d_1)\varepsilon_4+b_1(k_1+\varepsilon_1)\varepsilon^*]\leqslant\end{aligned}$$

$$r_1(k_1+\varepsilon_1)[-\varepsilon_0+b_1(k_1+\varepsilon_1)\varepsilon^*]<0$$

从而存在$\bar{\lambda}_2^*>0$,使得$I_2(\lambda)<0,\lambda\in(0,\bar{\lambda}_2^*)$,即有

$$\begin{aligned}&D_1\bar{\varphi}''(t)-c\bar{\varphi}'(t)r_1\bar{\varphi}(t)[1-a_1\bar{\varphi}(t)-\\&\quad b_1\bar{\varphi}(t-c\tau_1)-c_1\underline{\psi}(t)-\\&\quad d_1\underline{\psi}(t-c\tau_2)]\leqslant 0,\lambda\in(0,\bar{\lambda}_2^*)\end{aligned}$$

类似地,对任意的 $t\in\mathbb{R}$,存在$\bar{\lambda}_3^*>0$,使得对$\lambda\in(0,\bar{\lambda}_3^*)$时,有

$$\begin{aligned}&D_2\bar{\psi}''(t)-c\bar{\psi}'(t)+r_2\bar{\psi}(t)[1-a_2\bar{\psi}(t)-\\&\quad b_2\bar{\psi}(t-c\tau_3)-c_2\underline{\varphi}(t)-d_2\underline{\varphi}(t-c\tau_4)]\leqslant 0\end{aligned}$$

综上所述,取$\bar{\lambda}_4^*=\min\{\bar{\lambda}_1^*,\bar{\lambda}_2^*,\bar{\lambda}_3^*\}$,则对$\lambda\in(0,\bar{\lambda}_4^*)$,$\bar{\Phi}(t)=(\bar{\varphi}(t),\bar{\psi}(t))$是系统(10)的一个弱上解.

现证$\underline{\Phi}(t)=(\underline{\varphi}(t),\underline{\psi}(t))$是系统(10)的一个弱下解.

由 η 的定义,有 $\lambda_1<\eta\lambda_1<\lambda_2$ 和 $D_1\eta^2\lambda_1^2-c\eta\lambda_1+r_1<0$.

a. 当 $t\leqslant t_1$时,有 $t\leqslant t_4$,

$$\underline{\varphi}(t)=e^{\lambda_1 t}-q\,e^{\eta\lambda_1 t},$$

$$\underline{\varphi}(t-c\tau_1)=e^{\lambda_1(t-c\tau_1)}-q\,e^{\eta\lambda_1(t-c\tau_1)}$$

$$\bar{\psi}(t)=e^{\lambda_3 t}$$

$$\bar{\psi}(t-c\tau_2)=e^{\lambda_3(t-c\tau_2)}$$

因此

$$\begin{aligned}&D_1\underline{\varphi}''(t)-c\underline{\varphi}'(t)+r_1\underline{\varphi}(t)[1-a_1\underline{\varphi}(t)-\\&\quad b_1\underline{\varphi}(t-c\tau_1)-c_1\bar{\psi}(t)-d_1\bar{\psi}(t-c\tau_2)]=\\&\quad D_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})''-c(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})'+\\&\quad r_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})[1-a_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})-\\&\quad b_1(e^{\lambda_1(t-c\tau_1)}-qe^{\eta\lambda_1(t-c\tau_1)})-c_1e^{\lambda_3 t}-\\&\quad d_1e^{\lambda_3(t-c\tau_2)}]=D_1(\lambda_1^2e^{\lambda_1 t}-\eta^2\lambda_1^2qe^{\eta\lambda_1 t})-\\&\quad c(\lambda_1e^{\lambda_1 t}-\eta\lambda_1qe^{\eta\lambda_1 t})+r_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})-\\&\quad r_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})[a_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})+\\&\quad b_1(e^{\lambda_1(t-c\tau_1)}-qe^{\eta\lambda_1(t-c\tau_1)})+c_1e^{\lambda_3 t}+d_1e^{\lambda_3(t-c\tau_2)}]=\\&\quad e^{\lambda_1 t}(D_1\lambda_1^2-c\lambda_1+r_1)-qe^{\eta\lambda_1 t}(D_1\eta^2\lambda_1^2-\\&\quad c\eta\lambda_1+r_1)-r_1(e^{\lambda_1 t}-qe^{\eta\lambda_1 t})[a_1(e^{\lambda_1 t}-\\&\quad qe^{\eta\lambda_1 t})+b_1(e^{\lambda_1(t-c\tau_1)}-qe^{\eta\lambda_1(t-c\tau_1)})+c_1e^{\lambda_3 t}+\\&\quad d_1e^{\lambda_3(t-c\tau_2)}]\geqslant -qe^{\eta\lambda_1 t}(D_1\eta^2\lambda_1^2-c\eta\lambda_1+r_1)-\\&\quad r_1e^{\lambda_1 t}[a_1e^{\lambda_1 t}+b_1e^{\lambda_1(t-c\tau_1)}+c_1e^{\lambda_3 t}+\\&\quad d_1e^{\lambda_3(t-c\tau_2)}]\geqslant -qe^{\eta\lambda_1 t}(D_1\eta^2\lambda_1^2-c\eta\lambda_1+r_1)-\\&\quad r_1e^{\lambda_1 t}[(a_1+b_1)e^{\lambda_1 t}+(c_1+d_1)e^{\lambda_3 t}]=\\&\quad -qe^{\eta\lambda_1 t}[(D_1\eta^2\lambda_1^2-c\eta\lambda_1+r_1)+\frac{r_1}{q}(a_1+b_1)\cdot\\&\quad e^{(2\lambda_1-\eta\lambda_1)t}+\frac{r_1}{q}(c_1+d_1)e^{(\lambda_1+\lambda_3-\eta\lambda_1)t}]\geqslant -qe^{\eta\lambda_1 t}\cdot\\&\quad [(D_1\eta^2\lambda_1^2-c\eta\lambda_1+r_1)+\frac{r_1}{q}(a_1+b_1)\cdot\\&\quad e_1^{(2\lambda_1-\eta\lambda_1)t}+\frac{r_1}{q}(c_1+d_1)e^{(\lambda_1+\lambda_3-\eta\lambda_1)t_1}]\geqslant\\&\quad 0,q\text{ 充分大},t_1=\frac{1}{\eta\lambda_1-\lambda_1}\ln\frac{1}{q\eta}<0\end{aligned}$$

b. 当 $t\geqslant t_1+c\tau_1$时,

$$\underline{\varphi}(t)=k_1-\varepsilon_2e^{-\lambda t}$$

$$\underline{\varphi}(t-c\tau_1)=k_1-\varepsilon_2e^{-\lambda(t-c\tau_1)}$$

且对任意的 $t\in\mathbb{R}$,有 $0<\bar{\psi}(t)<k_2+\varepsilon_3$,$0<\bar{\psi}(t-c\tau_2)<k_2+\varepsilon_3$.于是,

$$\begin{aligned}&D_1\underline{\varphi}''(t)-c\underline{\varphi}'(t)+r_1\underline{\varphi}(t)[1-a_1\underline{\varphi}(t)-\\&\quad b_1\underline{\varphi}(t-c\tau_1)-c_1\bar{\psi}(t)-d_1\bar{\psi}(t-c\tau_2)]\geqslant\\&\quad D_1(k_1-\varepsilon_2e^{-\lambda t})''-c(k_1-\varepsilon_2e^{-\lambda t})'+\\&\quad r_1(k_1-\varepsilon_2e^{-\lambda t})[1-a_1(k_1-\varepsilon_2e^{-\lambda t})-\\&\quad b_1(k_1-\varepsilon_2e^{-\lambda(t-c\tau_1)})-(c_1+d_1)\cdot\\&\quad (k_2+\varepsilon_3)]\geqslant -D_1\varepsilon_2\lambda^2e^{-\lambda t}-c\varepsilon_2\lambda e^{-\lambda t}+\\&\quad r_1(k_1-\varepsilon_2e^{-\lambda t})[1-a_1(k_1-\varepsilon_2e^{-\lambda t})-\\&\quad b_1(k_1-\varepsilon_2e^{-\lambda(t-c\tau_1)})-\\&\quad (c_1+d_1)(k_2+\varepsilon_3)]=I_3(\lambda)\end{aligned}$$

从而

$$\begin{aligned}I_3(0)=&\,r_1(k_1-\varepsilon_2)[1-a_1(k_1-\varepsilon_2)-\\&b_1(k_1-\varepsilon_2)-(c_1+d_1)(k_2+\varepsilon_3)]=\\&r_1(k_1-\varepsilon_2)[1-(a_1+b_1)k_1-(c_1+\\&d_1)k_2+(a_1+b_1)\varepsilon_2-(c_1+d_1)\varepsilon_3]\geqslant\\&r_1(k_1-\varepsilon_2)\varepsilon_0>0\end{aligned}$$

因此,存在$\bar{\lambda}_5^*>0$,使得$I_3(\lambda)>0,\lambda\in(0,\bar{\lambda}_5^*)$,即

$$\begin{aligned}&D_1\underline{\varphi}''(t)-c\underline{\varphi}'(t)+r_1\underline{\varphi}(t)[1-a_1\underline{\varphi}(t)-\\&\quad b_1\underline{\varphi}(t-c\tau_1)-c_1\bar{\psi}(t)-\\&\quad d_1\bar{\psi}(t-c\tau_2)]\geqslant 0,\lambda\in(0,\bar{\lambda}_3^*)\end{aligned}$$

c. 当$t_1<t<t_1+c\tau_1$时,

$$\underline{\varphi}(t)=k_1-\varepsilon_2e^{-\lambda t}$$

$$\underline{\varphi}(t-c\tau_1)=e^{\lambda_1(t-c\tau_1)}-qe^{\eta\lambda_1(t-c\tau_1)}$$

对任意的 $t\in\mathbb{R}$,有 $0<\bar{\psi}(t)<k_2+\varepsilon_3$,$0<\bar{\psi}(t-c\tau_2)<k_2+\varepsilon_3$.则

$$\begin{aligned}&D_1\underline{\varphi}''(t)-c\underline{\varphi}'(t)+r_1\underline{\varphi}(t)[1-a_1\underline{\varphi}(t)-\\&\quad b_1\underline{\varphi}(t-c\tau_1)-c_1\bar{\psi}(t)-d_1\bar{\psi}(t-c\tau_2)]\geqslant\\&\quad D_1(k_1-\varepsilon_2e^{-\lambda t})''-c(k_1-\varepsilon_2e^{-\lambda t})'+\\&\quad r_1(k_1-\varepsilon_2e^{-\lambda t})[1-a_1(k_1-\varepsilon_2e^{-\lambda t})-\\&\quad b_1(e^{\lambda_1(t-c\tau_1)}-qe^{\eta\lambda_1(t-c\tau_1)})-(c_1+d_1)\cdot\\&\quad (k_2+\varepsilon_3)]=-D_1\varepsilon_2\lambda^2e^{-\lambda t}-c\varepsilon_2\lambda e^{-\lambda t}+\\&\quad r_1(k_1-\varepsilon_2e^{-\lambda t})[1-a_1(k_1-\varepsilon_2e^{-\lambda t})-\\&\quad b_1(e^{\lambda_1(t-c\tau_1)}-qe^{\eta\lambda_1(t-c\tau_1)})-(c_1+d_1)\cdot\end{aligned}$$

$$
\begin{aligned}
&(k_2+\varepsilon_3)] \geqslant -D_1\varepsilon_2\lambda^2 e^{-\lambda t} - c\varepsilon_2\lambda e^{-\lambda t} + \\
&r_1(k_1-\varepsilon_2 e^{-\lambda t})[1-a_1(k_1-\varepsilon_2 e^{-\lambda t}) - \\
&b_1(e^{\lambda_1 t_1} - q e^{\eta\lambda_1 t_1}) - (c_1+d_1)(k_2+\varepsilon_3)] = \\
&-D_1\varepsilon_2\lambda^2 e^{-\lambda t} - c\varepsilon_2\lambda e^{-\lambda t} + r_1(k_1-\varepsilon_2 e^{-\lambda t})\cdot \\
&[1-a_1(k_1-\varepsilon_2 e^{-\lambda t}) - b_1(k_1-\varepsilon_2 e^{-\lambda t_1}) - \\
&(c_1+d_1)(k_2+\varepsilon_3)] = I_4(\lambda)
\end{aligned}
$$

$$I_4(0) = I_3(0) > 0$$

因此，存在$\bar{\lambda}_6^* > 0$，当 $\lambda \in (0,\bar{\lambda}_6^*)$时，

$$
\begin{aligned}
&D_1\underline{\varphi}''(t) - c\underline{\varphi}'(t) + r_1\underline{\varphi}(t)[1-a_1\underline{\varphi}(t) - \\
&\quad b_1\underline{\varphi}(t-c\tau_1) - c_1\bar{\psi}(t) - d_1\bar{\psi}(t-c\tau_2)] \geqslant 0
\end{aligned}
$$

同理可证，对任意的 $t\in\mathbb{R}$，存在 $\bar{\lambda}_7^* > 0$，使得对 $\lambda\in(0,\bar{\lambda}_7^*)$有

$$
\begin{aligned}
&D_1\underline{\varphi}''(t) - c\underline{\varphi}'(t) + r_1\underline{\varphi}(t)[1-a_1\underline{\varphi}(t) - \\
&\quad b_1\underline{\varphi}(t-c\tau_1) - c_1\bar{\psi}(t) - d_1\bar{\psi}(t-c\tau_2)] \geqslant 0
\end{aligned}
$$

综上所述，取$\bar{\lambda}_8^* = \min\{\bar{\lambda}_5^*, \bar{\lambda}_6^*, \bar{\lambda}_7^*\}$，则存在 $\lambda\in(0,\bar{\lambda}_8^*)$，使得 $\underline{\Phi}(t) = (\underline{\varphi}(t), \underline{\psi}(t))$是系统(10)的一个弱下解.

引理 2 得证.

结合定理 1、引理 1 和引理 2，可得定理 2.

定理 2　假设$a_1+b_1 > c_2+d_2$，$a_2+b_2 > c_1+d_1$. 当 τ_1，$\tau_3 > 0$ 且充分小时，则对任意 $c > 2\max\{\sqrt{r_1D_1}, \sqrt{r_2D_2}\}$，系统(3)有连接平衡点 $E_1=(0,0)$和 $E_4=(k_1,k_2)$的行波解$(\varphi(x+ct)$，$\psi(x+ct))$，且有

$$\lim_{\xi\to-\infty}\varphi(t)e^{-\lambda_1\xi} = \lim_{\xi\to-\infty}\psi(t)e^{-\lambda_3\xi} = 1$$

参考文献：

[1] Kolmogorov A N. A study of the equation of diffusion with increase in the quantity of matter, and its application to a biological problem [J]. Moscow University Bulletin of Mathematics, 1937(1): 1－25.

[2] Van Vuuren J H. The existence of traveling plane waves in a general class of competition-diffusion systems [J]. IMA Journal of Applied Mathematics, 1995, 55(2): 135－148.

[3] Tang M M, File P C. Propagating fronts for competing species equations with diffusion [J]. Archive for Rational Mechanics and Analysis, 1980, 73 (1): 69－77.

[4] Gourley S A, Ruan S. Convergence and travelling fronts in functional differential equations with nonlocal terms: a competition model [J]. Siam Journal on Mathematical Analysis, 2003, 35(3): 806－822.

[5] Conley C, Gardner R. An application of the generalized Morse index to travelling wave solutions of a competitive reaction-diffusion model [J]. Indiana University mathematics Journal, 1984, 33(3): 319－343.

[6] Gardner R A. Existence and stability of travelling wave solutions of competition models: a degree theoretic approach [J]. Journal of Differential Equations, 1982, 44(3): 343－364.

[7] Kanon Y. Parameter dependence of propagation speed of travelling waves for competition-diffusion equations [J]. Siam Journal on Mathematical Analysis, 1995, 26 (2): 340－363.

[8] Kanel J, I, Zhou L. Existence of wave front solutions and estimates of wave speed for a competition-diffusion system [J]. Nonlinear Analysis: Theory, Methods & Applications, 1996, 27(5): 579－587.

[9] Lü G Y, Wang M X. Traveling wave front in diffusive and competitive Lotka-Volterra system with delays [J]. Nonlinear Analysis: Real World Applications, 2010, 11(3): 1323－1329.

[10] Li W T, Lin G, Ruan S G. Existence of traveling wave solutions in delayed reaction-diffusion systems with applications to diffusion-competition systems [J]. Nonlinearity, 2006, 19(6): 1253－1273.

[11] Xia J, Yu Z X, Traveling wave solutions in temporally discrete reaction-diffusion systems with delays [J]. Zamm-Journal of Applied Mathematics and Mechanics, 2011, 91(10): 809－823.

[12] Yu Z X, Yuan R. Traveling waves for a Lotka-Volterra competition system with diffusion [J]. Mathematical and Computer Modelling, 2011, 53(5/6): 1035－1043.

[13] Yu Z X, Yuan R. Traveling waves of delayed reaction diffusion systems with applications [J]. Nonlinear Analysis Real World Applications, 2011, 12 (5): 2475－2488.

[14] Yu Z X, Zhao H K. Traveling waves for competitive Lotka-Volterra systems with spatial diffusions and spatio-temporal delays [J]. Applied Mathematics and Computation, 2014, 242: 669－678.

[15] 刘艺昕，余志先. 格上时滞单种群模型的行波解的渐近性[J]. 上海理工大学学报，2015，37(4)：322－326.

[16] 夏静，余志先，袁荣. 一类具有非局部扩散的时滞 Lotka -Volterra 竞争模型的行波解[J]. 应用数学学报，2011，34(6)：1082－1093.

（编辑：石　瑛）