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射影空间上涉及 q,c 阶差分算子的第二基本定理

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摘要： 研究了从复平面$\mathbb{C}$到复射影空间$\mathbb{P}^n(\mathbb{C})$上零级全纯映射的第二基本定理问题。根据其他文献处理重值的方法和技巧，从 q,c 阶差分算子的角度得到了推广的 Nevanlinna 第二基本定理，即射影空间$\mathbb{P}^n$上零级全纯映射，涉及 q,c 阶差分算子 $\Delta_{q,c}f=f(qz+c)-f(z)$的第二基本定理. 并利用其证明了当 f 与 $\Delta_{q,c}f$ CM（计重数）分担至少 $N+2$ 个处于一般位置的超平面时，$W^{\Delta_{q,c}}(f)\equiv 0$. 改进和推广了一些处于一般位置超平面的全纯映射唯一性结果.

关键词： q,c 阶差分；全纯映射；超平面；朗斯基行列式
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Second Main Theorem About Holomorphic Maps on $\mathbb{P}^n(\mathbb{C})$ Related to q,c-Difference Operators

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Abstract: The second main theorem of holomorphic maps from $\mathbb{C}$ into $\mathbb{P}^n(\mathbb{C})$ of zero order was studied. According to the method and skill of other documents for processing heavy values, the second main theorem of generalized Nevanlinna was obtained from the view angle of difference operators, that is, the second main theorem about the holomorphic maps of zero-order on $\mathbb{P}^n(\mathbb{C})$, which are related to the q,c difference operator. By virtue of the theorem, it is proved that $W^{\Delta_{q,c}}(f)\equiv 0$, if f and $\Delta_{q,c}f$ share at least $N+2$ hyperplanes in general positions on $\mathbb{P}^n(\mathbb{C})$, and the results of holomorphic maps uniqueness are improved and generalized.

Keywords: *q, c difference operator; holomorphic maps; hyperplanes; Wronskian determinate*

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1　问题的提出

Nevanlinna[1] 在 1925 年建立了复平面上关于亚纯函数的第二基本定理. Cartan[2] 将其推广到复射影空间$\mathbb{P}^n(\mathbb{C})$中全纯曲线上，得到了相应的第二基本定理. Huang 等[3] 证明了 Nevanlinna 第二基本定理在复平面$\mathbb{C}$上更一般的形式，见定理 1。

定理 1　设函数 f 在复平面$\mathbb{C}$上非常值亚纯，$a_1,\cdots,a_q$ 是 q 个互不相同的复数，则对于任给的 $\varepsilon>0$ 有

$$\sum_{j=1}^{q} m_f(r,a_j)+N_{ram,f}(r)\leqslant 2T(r,f)+\log T_f(r)+(1+\varepsilon)\log^+\log T_f(r)+O(1)$$

式中，$N_{ram,f}(r)=N_{f'}(r,0)+2N_f(r,\infty)-N_{f'}(r,\infty)$，$r\to\infty$且 $r\notin E$，这里 $E\subset(0,\infty)$是一个对数测度为有穷的集合.

后来，Wong[4] 将这一经典理论推广到了高维的情况（更多的拓展参见文献[5－7]），得到如下定理.

定理 2[4]　设 $f:\mathbb{C}\to\mathbb{P}^n$ 是一个线性非退化的全纯映射，$H_1,\cdots,H_q$ 是 q 个处于一般位置的超平面，则

$$\sum_{j=1}^{q} m_f(r,H_j)+N_{ram,f}(r)\leqslant(n+1)T_f(r)+S(r,f)$$

式中，$N_{ram,f}(r)=N(r,\boldsymbol{W}(f)=0)$是 f 的朗斯基行列式零点的计数函数. $S(r,f)=o(T(r,f))$，$r\to\infty$且 $r\notin E$，这里 $E\subset(0,+\infty)$是一个对数测度为有穷的集合.

2006 年，Halburd 等[8] 考虑用差分算子 $\Delta_c f=f(z+c)-f(z)\not\equiv 0$ 来替代 f'，得到了涉及差分算子所对应的情形.

定理 3[8]　设 f 是复平面$\mathbb{C}$上的有穷级亚纯函数，满足 $\Delta_c f=f(z+c)-f(z)\not\equiv 0$，其中 $c\neq 0$ 是常数，则对于 q 个互异的点 $a_1,\cdots,a_q\in\mathbb{C}\cup\{\infty\}$有

$$\sum_{j=1}^{q} m_f(r,a_j)+N_{pair}(r)\leqslant 2T_f(r)+S(r,f)$$

其中

$$N_{pair}(r)=2N(r,f=\infty)-N(r,f(z+c)-f(z)=\infty)+N(r,f(z+c)-f(z)=0)$$

亚纯函数可视作复平面$\mathbb{C}$到射影空间$\mathbb{P}$的全纯映射，后来 Halburd 等[9] 又给出了差分朗斯基行列式 $\boldsymbol{W}^{\Delta_c}(f)$的定义，将定理 3 推广到高维复射影空间.

定理 4[9]　设 $f:\mathbb{C}\to\mathbb{P}^n$ 是一个有穷级的全纯映射，且 $f(\mathbb{C})\not\subset H_i$，$i=1,\cdots,q$，这里 $H_1,\cdots,H_q$ 是 q 个处于一般位置的超平面. 若 $\boldsymbol{W}^{\Delta_c}(f)\not\equiv 0$，其中 $c\neq 0$ 是常数，则

$$\sum_{j=1}^{q} m_f(r,H_j)+N_{\Delta_c}(r)\leqslant(n+1)T_f(r)+S(r,f)$$

式中，$N_{\Delta_c}(r)=N(r,\boldsymbol{W}^{\Delta_c}(f)=0)$.

受此启发，考虑如下的 q,c 阶差分算子 $\Delta_{q,c}f=f(qz+c)-f(z)$，其中 $q\in\mathbb{C}\setminus\{0\}$，$c\in\mathbb{C}$，并将定理 4 作如下推广，得到定理 5.

定理 5　设 $f:\mathbb{C}\to\mathbb{P}^n$ 是一个零级全纯映射，并且 $f(\mathbb{C})\not\subset H_i$，$i=1,\cdots,q$，$H_1,\cdots,H_q$ 是 q 个处于一般位置的超平面. 若 $\boldsymbol{W}^{\Delta_{q,c}}(f)\not\equiv 0$，$q\in\mathbb{C}\setminus\{0\}$，$c\in\mathbb{C}$，则对任给的 $\varepsilon>0$ 有

$$\sum_{j=1}^{q} m_f(r,H_j)+N_{\Delta_{q,c}}(r)\leqslant(n+1)T_f(r)+S(r,f)$$

式中，$N_{\Delta_{q,c}}(r)=N(r,\boldsymbol{W}^{\Delta_{q,c}}(f)=0)$.

2　引理及证明

引理 1　（Nevanlinna 第一基本定理）设 $f:\mathbb{C}\to\mathbb{P}^n$ 是亚纯映射，且 $f(\mathbb{C})\not\subset H$，这里 H 是处于一般位置的超平面，则

$$m_f(r,H)+N_f(r,H)=T_f(r)\quad(r>1)$$

引理 2[10]　设 f 是零级的亚纯函数，$q\in\mathbb{C}\setminus\{0\}$，$c\in\mathbb{C}$，则

$$m\left(r,\frac{f(qz+c)}{f(z)}\right)=S(r,f)$$

引理 3　设 f 是零级的亚纯函数，$q\in\mathbb{C}\setminus\{0\}$，$c\in\mathbb{C}$，则

$$m\left(r,\frac{f(z)}{f(qz+c)}\right)=S(r,f)$$

且 $T(r,qz+c)=T(r,f(z))+S(r,f)$.

定义 1　设 $f:\mathbb{C}\to\mathbb{P}^n(\mathbb{C})$是全纯映射，$\widetilde{f}=(f_0,\cdots,f_n)$是 f 的既约表示，其中 $f_0,f_1,\cdots,f_n$ 是无公共零点的全纯函数. 记 $H=\left\{[w_0,\cdots,w_n]\in\mathbb{P}^n(\mathbb{C})\,\middle|\,\sum_{i=0}^{n}a_iw_i=0\right\}$，其中 $\widetilde{a}=(a_0,\cdots,a_n)$是

$\mathbb{C}^{n+1}$上的非零表示. 当$<\widetilde{f}(z),\widetilde{\alpha}>\neq 0$时,记$\lambda_H(f(z))=\log\dfrac{\|\widetilde{f}(z)\|\,\|\widetilde{\alpha}\|}{|<\widetilde{f}(z),\widetilde{\alpha}>|}$, $m_f(r,H)=\int_0^{2\pi}\lambda_H(f(re^{i\theta}))\dfrac{d\theta}{2\pi}$.

类似于文献[10]中的定义,则有

定义 2 设 h 是亚纯函数,则 $\Delta_{q,c}^n h=\Delta_{q,c}^{n-1}(\Delta_{q,c}(h))$. 记 $h(z)\equiv h:=\overline{h}^{[0]}$, $h(qz+c)\equiv\overline{h}:=\overline{h}^{[1]}$, $h(q(qz+c)+c)\equiv\overline{\overline{h}}\equiv\overline{h}^{[2]}$, $\cdots$, $h(q(q\cdot(qz+c)+c)\cdots)\equiv\overline{h}^{[k]}$.

定义 3 设 $f:\mathbb{C}\to\mathbb{P}^n(\mathbb{C})$是全纯映射,$\widetilde{f}=(f_0,\cdots,f_n)$是 f 的既约表示,$\widetilde{\alpha}=(a_0,\cdots,a_n)$是$\mathbb{C}^{n+1}$上的非零表示,$\widetilde{f}$ 与 $\Delta_{q,c}\widetilde{f}$ 分担 $\widetilde{\alpha}$ 是指$<\widetilde{f},\widetilde{\alpha}>=0$当且仅当$<\Delta_{q,c}\widetilde{f},\widetilde{\alpha}>=0$. 若$<\widetilde{f},\widetilde{\alpha}>=0$与$<\Delta_{q,c}\widetilde{f},\widetilde{\alpha}>=0$有相同的零点和重数,则称$\widetilde{f}$ 与 $\Delta_{q,c}\widetilde{f}$ CM (counting multiplicities)分担 $\widetilde{\alpha}$.

引理 4[11] (转换律) 设 $f:\mathbb{C}\to\mathbb{P}^n$是一个全纯映射,$f=(f_0,\cdots,f_n)$,$\widetilde{f}$ 是 f 的一个既约表示,φ 是$\mathbb{P}^n$上的一个自同构,则

$$\boldsymbol{W}^{\Delta_{q,c}}(\varphi\circ f)=\boldsymbol{c}_\varphi\boldsymbol{W}^{\Delta_{q,c}}(f)$$

其中,

$$\boldsymbol{W}^{\Delta_{q,c}}(f)=\begin{vmatrix} f_0 & f_1 & \cdots & f_n\\ \Delta_{q,c}f_0 & \Delta_{q,c}f_1 & \cdots & \Delta_{q,c}f_n\\ \vdots & \vdots & & \vdots\\ \Delta_{q,c}^n f_0 & \Delta_{q,c}^n f_1 & \cdots & \Delta_{q,c}^n f_n\end{vmatrix},$$

$$q\in\backslash\{0\},c\in\mathbb{C}$$

$$\boldsymbol{c}_\varphi=\det\boldsymbol{\varphi}_*=\begin{vmatrix} a_{00} & a_{10} & \cdots & a_{n0}\\ a_{01} & a_{11} & \cdots & a_{n1}\\ \vdots & \vdots & & \vdots\\ a_{0n} & a_{1n} & \cdots & a_{nn}\end{vmatrix}\neq 0$$

证明 记

$$g_j=L_j\circ\widetilde{f}$$

其中 $L_j(w)=\sum_{i=0}^n a_{ji}w_i$, $j=0,1,\cdots,n$, $w=(w_0,\cdots,w_n)\in\mathbb{C}^{n+1}\backslash\{0\}$,$(w_0,\cdots,w_n)$是$\mathbb{P}^n$上的一组齐次坐标. 则

$$\boldsymbol{W}^{\Delta_{q,c}}(\varphi\circ f)=\begin{vmatrix} g_0 & g_1 & \cdots & g_n\\ \Delta_{q,c}g_0 & \Delta_{q,c}g_1 & \cdots & \Delta_{q,c}g_n\\ \vdots & \vdots & & \vdots\\ \Delta_{q,c}^n g_0 & \Delta_{q,c}^n g_1 & \cdots & \Delta_{q,c}^n g_n\end{vmatrix}=$$

$$\left|\begin{bmatrix} f_0 & f_1 & \cdots & f_n\\ \Delta_{q,c}f_0 & \Delta_{q,c}f_1 & \cdots & \Delta_{q,c}f_n\\ \vdots & \vdots & & \vdots\\ \Delta_{q,c}^n f_0 & \Delta_{q,c}^n f_1 & \cdots & \Delta_{q,c}^n f_n\end{bmatrix}\begin{bmatrix} a_{00} & a_{10} & \cdots & a_{n0}\\ a_{01} & a_{11} & \cdots & a_{n1}\\ \vdots & \vdots & & \vdots\\ a_{0n} & a_{1n} & \cdots & a_{nn}\end{bmatrix}\right|=$$

$$\begin{vmatrix} f_0 & f_1 & \cdots & f_n\\ \Delta_{q,c}f_0 & \Delta_{q,c}f_1 & \cdots & \Delta_{q,c}f_n\\ \vdots & \vdots & & \vdots\\ \Delta_{q,c}^n f_0 & \Delta_{q,c}^n f_1 & \cdots & \Delta_{q,c}^n f_n\end{vmatrix}\begin{vmatrix} a_{00} & a_{10} & \cdots & a_{n0}\\ a_{01} & a_{11} & \cdots & a_{n1}\\ \vdots & \vdots & & \vdots\\ a_{0n} & a_{1n} & \cdots & a_{nn}\end{vmatrix}=$$

$$\boldsymbol{W}^{\Delta_{q,c}}(f)\det\boldsymbol{\varphi}_*=\boldsymbol{W}^{\Delta_{q,c}}(f)\boldsymbol{c}_\varphi$$

于是引理 4 得证.

引理 5 设 $H_1,\cdots,H_q$ 是$\mathbb{P}^n(\mathbb{C})$上 q 个处于一般位置的超平面,记 T 为所有单射 $\mu:\{0,1,\cdots,n\}\to\{1,\cdots,q\}$的集合,则有

$$\sum_{j=1}^q m_f(r,H_j)\leqslant$$

$$\int_0^{2\pi}\max_{\mu\in T}\sum_{i=0}^n\lambda_{H_{\mu(i)}}(f(re^{i\theta}))\frac{d\theta}{2\pi}+S(r,f)$$

类似于文献[10]中定理 A3.1.3,则有

引理 6 设 $f=[f_0,\cdots,f_n]:\mathbb{C}\to\mathbb{P}^n(\mathbb{C})$是零级的全纯曲线,$H_1,\cdots,H_q$ 是$\mathbb{P}^n(\mathbb{C})$上任意 q 个超平面,且使得 $f(\mathbb{C})\not\subset H_i$, $i=1,\cdots,q$. 设 $K\subset\{1,\cdots,q\}$,使得 $\widetilde{\alpha}_k$($k\in K$,$\widetilde{\alpha}_1,\cdots,\widetilde{\alpha}_q$ 是 H_j($1\leqslant j\leqslant q$)的系数)线性无关. 则

$$\int_0^{2\pi}\max_K\sum_{k\in K}\lambda_{H_k}(f(re^{i\theta}))\frac{d\theta}{2\pi}+$$

$$N(r,\boldsymbol{W}^{\Delta_{q,c}(f)}=0)\leqslant(n+1)T_f(r)+S(r,f)$$

$r\to\infty$且 $r\notin E$,这里 $E\subset(0,\infty)$是一个对数测度为有穷的集合.

证明 设 $H_1,\cdots,H_q$ 是$\mathbb{C}^{n+1}$上的超平面,不失一般性,$q\geqslant n+1$, $\# K=n+1$. 记 T 是所有使得$\widetilde{\alpha}_{\mu(0)},\cdots,\widetilde{\alpha}_{\mu(n)}$线性无关的单射 $\mu:\{0,1,\cdots,n\}\to\{1,\cdots,q\}$的集合. 则

$$\int_0^{2\pi}\max_K\sum_{k\in K}\lambda_{H_k}(f(re^{i\theta}))\frac{d\theta}{2\pi}=\int_0^{2\pi}\max_{\mu\in T}\sum_{j=0}^n\log\left(\frac{\|\widetilde{f}(re^{i\theta})\|\,\|\widetilde{\alpha}_{\mu(j)}\|}{|<\widetilde{f}(re^{i\theta}),\widetilde{\alpha}_{\mu(j)}>|}\right)\frac{d\theta}{2\pi}=$$

$$\int_0^{2\pi}\log\left\{\max_{\mu\in T}\left(\frac{\|\widetilde{f}(re^{i\theta})\|^{n+1}}{\prod_{j=0}^{n}|<\widetilde{f}(re^{i\theta}),\widetilde{a}_{\mu(j)}>|}\right)\right\}\frac{d\theta}{2\pi}+O(1)\leqslant$$

$$\int_0^{2\pi}\log\left\{\sum_{\mu\in T}\left(\frac{\|\widetilde{f}(re^{i\theta})\|^{n+1}}{\prod_{j=0}^{n}|<\widetilde{f}(re^{i\theta}),\widetilde{a}_{\mu(j)}>|}\right)\right\}\frac{d\theta}{2\pi}+O(1)=$$

$$\int_0^{2\pi}\log\sum_{\mu\in T}\left\{\left(\frac{|W^{\Delta_{q,c}}(<\widetilde{f},\widetilde{a}_{\mu(0)}>,\cdots,<\widetilde{f},\widetilde{a}_{\mu(n)}>)(re^{i\theta})|}{\prod_{j=0}^{n}|<\widetilde{f}(re^{i\theta}),\widetilde{a}_{\mu(j)}>|}\right)\cdot\right.$$

$$\left.\left(\frac{\|\widetilde{f}(re^{i\theta})\|^{n+1}}{|W^{\Delta_{q,c}}(<\widetilde{f},\widetilde{a}_{\mu(0)}>,\cdots,<\widetilde{f},\widetilde{a}_{\mu(n)}>)(re^{i\theta})|}\right)\right\}\frac{d\theta}{2\pi}+O(1)=$$

$$\int_0^{2\pi}\log\left\{\sum_{\mu\in T}\left(\frac{|W^{\Delta_{q,c}}(<\widetilde{f},\widetilde{a}_{\mu(0)}>,\cdots,<\widetilde{f},\widetilde{a}_{\mu(n)}>)(re^{i\theta})|}{\prod_{j=0}^{n}|<\widetilde{f}(re^{i\theta}),\widetilde{a}_{\mu(j)}>|}\right)\right\}\frac{d\theta}{2\pi}+$$

$$\int_0^{2\pi}\log\left(\frac{\|\widetilde{f}(re^{i\theta})\|^{n+1}}{|W^{\Delta_{q,c}}(<\widetilde{f},\widetilde{a}_{\mu(0)}>,\cdots,<\widetilde{f},\widetilde{a}_{\mu(n)}>)(re^{i\theta})|}\right)\frac{d\theta}{2\pi}+O(1):=I_1+I_2+O(1)$$

记 $h_{\mu(l)}=<\widetilde{f},\widetilde{a}_{\mu(l)}>,g_{\mu(l)}=\dfrac{h_{\mu(l)}}{h_{\mu(0)}},0\leqslant l\leqslant n$,则 $g_{\mu(0)}=1$,显然 $T_{g_{\mu(l)}}\leqslant T_f(r)+O(1)$.因此

$$I_1=\int_0^{2\pi}\log\left\{\sum_{\mu\in T}\left(\frac{|W^{\Delta_{q,c}}(<\widetilde{f},\widetilde{a}_{\mu(0)}>,\cdots,<\widetilde{f},\widetilde{a}_{\mu(n)}>)|(re^{i\theta})}{\prod_{j=0}^{n}|<\widetilde{f}(re^{i\theta}),\widetilde{a}_{\mu(j)}>|}\right)\right\}\frac{d\theta}{2\pi}=$$

$$\int_0^{2\pi}\log\sum_{\mu\in T}\left(\frac{|W^{\Delta_{q,c}}(h_{\mu(0)},h_{\mu(1)},\cdots,h_{\mu(n)})|}{|h_{\mu(0)}h_{\mu(1)}\cdots h_{\mu(n)}|}(re^{i\theta})\right)\frac{d\theta}{2\pi}=$$

$$\int_0^{2\pi}\left\{\log\sum_{\mu\in T}\left(\frac{|W^{\Delta_{q,c}}(h_{\mu(0)},h_{\mu(1)},\cdots,h_{\mu(n)})|}{|h_{\mu(0)}\overline{h}_{\mu(1)}\cdots\overline{h}_{\mu(n)}^{[n]}|}\right)\left(\frac{|h_{\mu(0)}\overline{h}_{\mu(1)}\cdots\overline{h}_{\mu(n)}^{[n]}|}{|h_{\mu(0)}h_{\mu(1)}\cdots h_{\mu(n)}|}\right)(re^{i\theta})\right\}\frac{d\theta}{2\pi}=$$

$$\int_0^{2\pi}\left\{\log\sum_{\mu\in T}\left(\frac{|W^{\Delta_{q,c}}(h_{\mu(0)},h_{\mu(1)},\cdots,h_{\mu(n)})|}{|h_{\mu(0)}\overline{h}_{\mu(1)}\cdots\overline{h}_{\mu(n)}^{[n]}|}\right)(re^{i\theta})\right\}\frac{d\theta}{2\pi}+$$

$$\int_0^{2\pi}\left\{\log\left(\frac{|h_{\mu(0)}\overline{h}_{\mu(1)}\cdots\overline{h}_{\mu(n)}^{[n]}|}{|h_{\mu(0)}h_{\mu(1)}\cdots h_{\mu(n)}|}\right)(re^{i\theta})\right\}\frac{d\theta}{2\pi}\leqslant$$

$$\sum_{\mu\in T}\int_0^{2\pi}\left\{\log\left(\frac{|W^{\Delta_{q,c}}(h_{\mu(0)},h_{\mu(1)},\cdots,h_{\mu(n)})|}{|h_{\mu(0)}\overline{h}_{\mu(1)}\cdots\overline{h}_{\mu(n)}^{[n]}|}\right)(re^{i\theta})\right\}\frac{d\theta}{2\pi}+$$

$$\int_0^{2\pi}\left\{\log\left(\frac{|h_{\mu(0)}\overline{h}_{\mu(1)}\cdots\overline{h}_{\mu(n)}^{[n]}|}{|h_{\mu(0)}h_{\mu(1)}\cdots h_{\mu(n)}|}\right)(re^{i\theta})\right\}\frac{d\theta}{2\pi}+O(1)$$

其中

$$\frac{|W^{\Delta_{q,c}}(h_{\mu(0)},h_{\mu(1)},\cdots,h_{\mu(n)})|}{|h_{\mu(0)}\overline{h}_{\mu(1)}\cdots\overline{h}_{\mu(n)}^{[n]}|}=\frac{\begin{vmatrix}1&g_{\mu(1)}&\cdots&g_{\mu(n)}\\1&\overline{g}_{\mu(1)}&\cdots&\overline{g}_{\mu(n)}\\\vdots&\vdots&&\vdots\\1&\overline{g}_{\mu(1)}^{[n]}&\cdots&\overline{g}_{\mu(n)}^{[n]}\end{vmatrix}}{|\overline{g}_{\mu(1)}\overline{g}_{\mu(2)}^{[2]}\cdots\overline{g}_{\mu(n)}^{[n]}|}=\frac{\begin{vmatrix}1&1&\cdots&1\\1&\dfrac{\overline{g}_{\mu(1)}}{g_{\mu(1)}}&\cdots&\dfrac{\overline{g}_{\mu(n)}}{g_{\mu(n)}}\\\vdots&\vdots&&\vdots\\1&\dfrac{\overline{g}_{\mu(1)}^{[n]}}{g_{\mu(1)}}&\cdots&\dfrac{\overline{g}_{\mu(n)}^{[n]}}{g_{\mu(n)}}\end{vmatrix}}{\left|\dfrac{\overline{g}_{\mu(1)}}{g_{\mu(1)}}\dfrac{\overline{g}_{\mu(2)}^{[2]}}{g_{\mu(2)}}\cdots\dfrac{\overline{g}_{\mu(n)}^{[n]}}{g_{\mu(n)}}\right|}\leqslant$$

$$\frac{\displaystyle\sum_{i_1+\cdots+i_n\leqslant\frac{n(n+1)}{2}}\left|\frac{\overline{g}^{[i_1]}_{\mu(1)}}{g_{\mu(1)}}\cdots\frac{\overline{g}^{[i_n]}_{\mu(n)}}{g_{\mu(n)}}\right|}{\left|\frac{\overline{g}_{\mu(1)}}{g_{\mu(1)}}\frac{\overline{g}^{[2]}_{\mu(2)}}{g_{\mu(2)}}\cdots\frac{\overline{g}^{[n]}_{\mu(n)}}{g_{\mu(n)}}\right|}=\frac{\displaystyle\sum_{i_1+\cdots+i_n\leqslant\frac{n(n+1)}{2}}\sum_{l=1}^{n}\left|\frac{\overline{g}^{[i_l]}_{\mu(l)}}{g_{\mu(l)}}\right|}{\left|\frac{\overline{g}_{\mu(1)}}{g_{\mu(1)}}\frac{\overline{g}^{[2]}_{\mu(2)}}{g_{\mu(2)}}\cdots\frac{\overline{g}^{[n]}_{\mu(n)}}{g_{\mu(n)}}\right|}$$

结合引理2和引理3可知

$$\int_0^{2\pi}\log^+\left|\frac{g_{\mu(l)}}{\overline{g}^{[l]}_{\mu(l)}}\right|(re^{i\theta})\frac{d\theta}{2\pi}\leqslant S(r,f)$$

$$\int_0^{2\pi}\log^+\left|\frac{\overline{g}^{[l]}_{\mu(l)}}{g_{\mu(l)}}\right|(re^{i\theta})\frac{d\theta}{2\pi}\leqslant S(r,f)$$

$$\int_0^{2\pi}\log^+\left|\frac{\overline{h}^{[l]}_{\mu(l)}}{h_{\mu(l)}}\right|(re^{i\theta})\frac{d\theta}{2\pi}\leqslant S(r,f)$$

故

$$\sum_{\mu\in T}\int_0^{2\pi}\left\{\log\left(\frac{|\boldsymbol{W}^{\Delta_{q,c}}(h_{\mu(0)},h_{\mu(1)},\cdots,h_{\mu(n)})|}{|h_{\mu(0)}\overline{h}_{\mu(1)}\cdots\overline{h}^{[n]}_{\mu(n)}|}\right)(re^{i\theta})\right\}\frac{d\theta}{2\pi}\leqslant$$

$S(r,f)$.

且

$$\int_0^{2\pi}\log\left\{\left(\frac{|h_{\mu(0)}\overline{h}_{\mu(1)}\cdots\overline{h}^{[n]}_{\mu(n)}|}{|h_{\mu(0)}h_{\mu(1)}\cdots h_{\mu(n)}|}\right)(re^{i\theta})\right\}\frac{d\theta}{2\pi}\leqslant$$

$$\sum_{l=1}^{n}\int_0^{2\pi}\log\left|\frac{\overline{h}^{[l]}_{\mu(l)}}{h_{\mu(l)}}\right|(re^{i\theta})\frac{d\theta}{2\pi}\leqslant S(r,f)$$

所以

$$I_1\leqslant S(r,f)$$

又

$$I_2=\int_0^{2\pi}\log\left\{\frac{\|\widetilde{f}\|^{n+1}}{|\boldsymbol{W}^{\Delta_{q,c}}(<\widetilde{f}_0,\widetilde{\alpha}_{\mu(0)}>,\cdots,<\widetilde{f}_n,\widetilde{\alpha}_{\mu(n)}>)(re^{i\theta})|}\right\}\frac{d\theta}{2\pi}$$

由引理4可得

$$|\boldsymbol{W}^{\Delta_{q,c}}(<\widetilde{f}_0,\widetilde{\alpha}_{\mu(0)}>,\cdots,<\widetilde{f}_n,\widetilde{\alpha}_{\mu(n)}>)|=C|\boldsymbol{W}^{\Delta_{q,c}}(f_0,\cdots,f_n)|$$

这里$C\neq0$是常数,则

$$I_2=\int_0^{2\pi}\log\left\{\frac{\|\widetilde{f}\|^{n+1}}{|\boldsymbol{W}^{\Delta_{q,c}}(<\widetilde{f}_0,\widetilde{\alpha}_{\mu(0)}>,\cdots,<\widetilde{f}_n,\widetilde{\alpha}_{\mu(n)}>)(re^{i\theta})|}\right\}\frac{d\theta}{2\pi}=$$

$$\int_0^{2\pi}\log\|\widetilde{f}\|^{n+1}\frac{d\theta}{2\pi}+\int_0^{2\pi}\log\frac{1}{|\boldsymbol{W}^{\Delta_{q,c}}(f_0,\cdots,f_n)|}\frac{d\theta}{2\pi}+O(1)=$$

$$(n+1)T_f(r)-N(r,\boldsymbol{W}^{\Delta_{q,c}}(f)=0)+O(1).$$

结合I_1,I_2,从而引理6可证.

3 定理5的证明

结合引理5和引理6可得

$$\int_0^{2\pi}\max_K\sum_{k\in K}\lambda_{H_k}(f(re^{i\theta}))\frac{d\theta}{2\pi}+N(r,\boldsymbol{W}^{\Delta_{q,c}}(f)=0)\leqslant$$

$$(n+1)T_f(r)+S(r,f)$$

$$\sum_{j=1}^{q}m_f(r,H_j)\leqslant$$

$$\int_0^{2\pi}\max_{\mu\in T}\sum_{i=0}^{n}\lambda_{H_{\mu(i)}}(f(re^{i\theta}))\frac{d\theta}{2\pi}+O(1)$$

两式相加可得

$$\sum_{j=1}^{q}m_f(r,H_j)+N(r,\boldsymbol{W}^{\Delta_{q,c}}(f)=0)\leqslant$$

$$(n+1)T_f(r)+S(r,f)$$

于是定理得证.

推论 设$f:\mathbb{C}\to\mathbb{P}^n$是一个零级全纯映射,并且$f(\mathbb{C})\not\subset H_i$,$i=1,\cdots,q$,$H_1,\cdots,H_q$是$q$个处于一般位置的超平面,$f$与$\Delta_{q,c}f$ CM分担H_j.若$q\geqslant N+2$,则$\boldsymbol{W}^{\Delta_{q,c}}(f)\equiv0$.

证明 倘若不然,可假设$\boldsymbol{W}^{\Delta_{q,c}}(f)\equiv0$.由定理5可得

$$\sum_{j=1}^{q}m_f(r,H_j)+N(r,\boldsymbol{W}^{\Delta_{q,c}}(f)=0)\leqslant$$

$$(n+1)T_f(r)+S(r,f)$$

结合第一基本定理得

$$(q-n-1)T_f(r)\leqslant\sum_{j=1}^{q}N_f(r,H_j)-$$

$$N(r,\boldsymbol{W}^{\Delta_{q,c}}(f)=0)+S(r,f)$$

设$z_0\in\mathbb{C}$,$<f,H_j>(z_0)=0$重数为$m(m>1)$,又f与$\Delta_{q,c}f$ CM分担$H_j(j=1,\cdots,q)$,z_0作为$<\Delta^l_{q,c}(f),H_j>$的零点,重数也为$m(m>1)$,$l=1,\cdots,n$,所以z_0作为$\boldsymbol{W}^{\Delta_{q,c}}(f)$的零点,重数至少为$m$.故

$$\sum_{j=1}^{q}N_f(r,H_j)-N(r,\boldsymbol{W}^{\Delta_{q,c}}(f)=0)\leqslant0$$

所以$(q-n-1)T_f(r)\leqslant0$.又$q>N+2$,故得出矛盾.因此

$$\boldsymbol{W}^{\Delta_{q,c}}(f)\equiv0.$$

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