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涉及周期移动超平面的全纯曲线差分形式的第二基本定理

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摘要：利用全纯映射的值分布理论和对数导数引理，在已有的处于次一般位置的全纯曲线关于差分算子的第二基本定理的基础上，研究了涉及逐点处于次一般位置的周期移动超平面的全纯曲线关于差分算子的第二基本定理，推广了第二基本定理并得到了相应的结论.

关键词：全纯曲线；Nochka 权重；N-subgeneral 位置；周期移动超平面
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A Second Main Theorem for Holomorphic Curves Intersecting Periodic Moving Hyperplanes Concerning Difference Operators

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Abstract: Using the holomorphic mapping value distribution theory and the logarithmic derivative lemma and on the basis of the predecessors' second main theorems concerning differential operators, where the holomorphic curves are in sub general positions, the second main theorem for holomorphic curves intersecting periodic moving hyperplanes concerning difference operators in pointwise N-subgeneral position, was put forward. The second main theorem was generalized and the corresponding conclusions were presented.

Keywords: *holomorphic curve*; *Nochka weight*; *N-subgeneral position*; *periodic moving hyperplane*

1　问题的提出

Nevanlinna[1]在 1925 年建立了复平面上关于亚纯函数的第二基本定理。文献[2]将其推广到了复射影空间 $\mathbb{P}^n(\mathbb{C})$ 中的全纯曲线上，得到了相应的第二基本定理。2006 年，Halburd 等[3]证明了有穷级全纯映射涉及差分算子的第二基本定理。

2016 年，Cao 等[4]证明了在 $\mathbb{P}^n(\mathbb{C})$中超级 $\varsigma_2(f)=\varsigma<1$ 的亚纯映射，取逐点处于 N-subgeneral 位置的超平面$H_j(1\leqslant j\leqslant q)$。

现考虑用周期移动超平面$H_j(z)$代替超平面H_j，得到定理 1。

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定理 1　设 $Q=\{1,2,\cdots,q\}$，$f:\mathbb{C}\to\mathbb{P}^n(\mathbb{C})$为$\mathcal{P}_c$中线性非退化的全纯曲线，且$\varsigma_2(f)=\varsigma<1$，$H_j(z)$为$q(q>2N-n+1)$个在$\mathbb{P}^n(\mathbb{C})$中逐点处于 N-subgeneral 位置的周期移动超平面，则有

$$(q-2N+n-1)T_f(r)\leqslant\sum_{j\in Q}N_f(r,H_j(z))-\frac{N+1}{n+1}N(r,W^{\Delta_c}(f)=0)+o(T_f(r))+o\left(\frac{T_f(r)}{r^{1-\varsigma-\varepsilon}}\right)$$

其中，$r>0,r\notin E,\int_E\frac{\mathrm{d}t}{t}<\infty$。

2　符号与定义

定义 1[3-4]　设$f:\mathbb{C}\to\mathbb{P}^n(\mathbb{C})$为全纯曲线，则称

$$\varsigma(f):=u\limsup_{r\to\infty}\frac{\log^+T_f(r)}{\log r}$$

$$\varsigma_2(f):=\limsup_{r\to\infty}\frac{\log^+\log^+T_f(r)}{\log r}$$

为$f(z)$的级和超级。

$$T_f(r)=m_f(r,H(z))+N_f(r,H(z))+O(1)=\int_0^{2\pi}\log\left\|f(re^{i\theta})\right\|\frac{\mathrm{d}\theta}{2\pi}+O(1)$$

是$f(z)$的特征函数。其中，

$$m_f(r,H(z))=\int_0^{2\pi}\log^+\frac{\left\|f(re^{i\theta})\right\|\left\|H(re^{i\theta})\right\|}{\left|(f(re^{i\theta}),H(re^{i\theta}))\right|}\frac{\mathrm{d}\theta}{2\pi}$$

$$N_f(r,H_j(z))=\int_0^{2\pi}\log\left|(f(re^{i\theta}),H_j(re^{i\theta}))\right|\frac{\mathrm{d}\theta}{2\pi}+O(1)$$

定义 2[3]　$\mathcal{P}_c=\{a(z)$在$\mathbb{C}$上亚纯，$a(z+c)=a(z)$，$T(r,a(z))=o(T_f(r))\}$。

定义 3[5]　称$H_j(z):=\left\{[w_0:\cdots:w_n]:\sum_{i=0}^{q}a_{ji}(z)w_i=0\right\}$为周期移动的超平面，其中，$[w_0:\cdots:w_n]$为$\mathbb{P}^n(\mathbb{C})$中齐次坐标，$a_{ji}(z)\in\mathcal{P}_c$。

定义 4[3]　若对任意集合$R\subset Q=\{1,2,\cdots,q\}$且$\#R=N+1\geqslant n+1$，都有$\bigcap_{j\in R}H_j(z)=\varnothing$，对所有的$z\in\mathbb{C}$成立，称$H_j(z)(j\in Q)$在$\mathbb{P}^n(\mathbb{C})$中逐点处于 N-subgeneral 位置。

定义 5　称$\Delta_cf(z):=f(z+c)-f(z)$为$c-$差分算子，$\Delta_c^n:=\Delta_c^{n-1}(\Delta_cf)$为$n$阶$c-$差分算子，称

$$W^{\Delta_c}(f)=\begin{vmatrix}f_0&f_1&\cdots&f_n\\\Delta_cf_0&\Delta_cf_1&\cdots&\Delta_cf_n\\\vdots&\vdots&&\vdots\\\Delta_c^nf_0&\Delta_c^nf_1&\cdots&\Delta_c^nf_n\end{vmatrix}$$

为差分 Wronskian 行列式。

3　主要引理

引理 1[2,6]　设 $Q=\{1,2,\cdots,q\}$，$H_j(z)(j\in Q)$为$\mathbb{P}^n(\mathbb{C})$中逐点处于 N-subgeneral 位置的移动超平面，$q>2N-n+1$，则存在有理数$\omega(j,z)>0$ $(j\in Q)$和$\tilde{\omega}(z)>0$，满足下列性质：

a. $0<\frac{\omega(j,z)}{\tilde{\omega}(z)}\leqslant 1\ (j\in Q)$；

b. $\sum_{j=1}^{q}\omega(j,z)=\tilde{\omega}(z)(q-2N+n-1)+n+1$；

c. $\frac{n+1}{2N-n+1}\leqslant\tilde{\omega}(z)\leqslant\frac{n+1}{N+1}$；

d. 对$R\subset Q$且$0<\#R\leqslant N+1$，有$\sum_{j\in R}\omega(j,z)\leqslant d(R)$；

e. $E_j\geqslant 1\ (j\in Q)$为任意给定的数，则对每一个子集$R\subset Q$且$0<\#R\leqslant N+1$，存在子集$R^0\subset R$且$\#R^0=d(R^0)=d(R)$，则有

$$\prod_{j\in R}E_j^{\omega(j,z)}\leqslant\prod_{j\in R^0}E_j$$

式中，$\omega(j,z)$和$\tilde{\omega}(z)$为相应的 Nochka 权重和 Nochka 常数。

引理 2　设$q>2N-n+1$，$Q=\{1,2,\cdots,q\}$，若$f:\mathbb{C}\to\mathbb{P}^n(\mathbb{C})$为$\mathcal{P}_c$中线性非退化的全纯曲线，$\{H_j(z)\}_{j=1}^q$为$\mathbb{P}^n(\mathbb{C})$中逐点处于 N-subgeneral 位置的周期移动超平面，$\omega(j,z)$和$\tilde{\omega}(z)$为相应的 Nochka 权重和 Nochka 常数，则对任意$z\in\mathbb{C}\backslash\left(z\in\mathbb{C}:\left(\prod_{t_j\in R}\left|(f(z+jc),H_{t_j}(z))\right|\right)=0\right)$，有

$$\|f\|^{\tilde{\omega}(z)(q-2N+n-1)}\leqslant K\frac{\left(\prod_{t_j\in R}\left|(f(z+jc),H_{t_j}(z))\right|^{\omega(t_j,z)}\right)\left(\prod_{j\in s}\left|(f(z),H_j(z))\right|^{\omega(j,z)}\right)}{\left|\prod_{i\in 0}^{n}(f(z+ic),H_{t_i}(z))\right|}$$

这里，$R=\{t_0,\cdots,t_n,t_{n+1},\cdots,t_N\}\subset Q$，$R^0=\{t_0,\cdots,t_n\}\subset R$，$S=Q\backslash R$，其中，$K>0$，$K$是与$j$有关的常数。

证明　因为移动超平面$\{H_j(z)\}_{j=1}^q$在$\mathbb{P}^n(\mathbb{C})$中逐点处于 N-subgeneral 位置，所以，对任意$R\subset Q$且$\#R=N+1$，有$\bigcap_{j\in R}H_j(z)=\varnothing$，则存在$S=Q\backslash R\subset Q$且$\#S=q-N-1$，使得$\prod_{j\in S}(f(z),H_j(z))\neq 0$，对任意$z\in\mathbb{C}\backslash\left(z\in\mathbb{C}:\bigcup_{j\in Q}(f(z+k_jc),H_j(z))=0\right)$，有$K_{k_j}(z)(K_{k_j}(z)$

的取值与$H_j(z)$和$k_j\in N^+\bigcup\{0\}$有关)，使得

$$\frac{1}{|K_{k_j}(z)|}\leqslant\frac{|(f(z+k_jc),H_j(z))|}{\|f(z)\|}\leqslant|K_{k_j}(z)|,\ j\in S$$

取适当的r，使得$\{z:|z|=r\}\subset\mathbb{C}\setminus\left(z\in\mathbb{C}:\bigcup_{j\in Q}(f(z+k_jc),H_j(z))=0\right)$，由有限覆盖定理，存在$K_{k_j}$($K_{k_j}$取值与$k_j\in N^+\bigcup\{0\}$有关)，使得

$$\frac{1}{|K_{k_j}|}\leqslant\frac{|(f(z+k_jc),H_j(z))|}{\|f(z)\|}\leqslant|K_{k_j}|,\ j\in S \tag{1}$$

令$R=Q\setminus S=\{t_0,\cdots,t_n,t_{n+1},\cdots,t_N\}$，则有$\#R=N+1$，$d(R)=n+1$。

由引理 1 的性质 e，取$E_j=\dfrac{\|f(z)\||K_{k_j}|}{|(f(z+k_j),H_j(z))|}\geqslant1$，则存在$R^0=\{t_0,\cdots,t_n\}\subset R$，$\#R^0=d(R^0)=d(R)$，使得

$$\prod_{j\in R}\left(\frac{\|f(z)\||K_{k_j}|}{|(f(z+k_jc),H_j(z))|}\right)^{\omega(j,z)}\leqslant\prod_{j\in R^0}\left(\frac{\|f(z)\||K_{k_j}|}{|(f(z+k_jc),H_j(z))|}\right) \tag{2}$$

对任意 $z\in\mathbb{C}\setminus\left(z\in\mathbb{C}:\left(\prod_{t_j\in R}|(f(z+jc),H_{t_j}(z))|\right)=0\right)$。

在式(1)中取$k_j=0$，可得

$$\prod_{j\in S}\left(\frac{1}{|K_j|^2}\right)^{\omega(j,z)}\leqslant\prod_{j\in S}\left(\frac{|(f(z),H_j(z))|}{\|(f(z)\||K_j|}\right)^{\omega(j,z)}\leqslant\prod_{t_j\in R}\left(\frac{\|f(z)\||K_{t_j}|}{|(f(z+jc),H_{t_j}(z))|}\right)^{\omega(t_j,z)}\cdot$$
$$\frac{\left(\prod_{t_j\in R}|(f(z+jc),H_{t_j}(z))|^{\omega(t_j,z)}\right)\left(\prod_{j\in S}|(f(z),H_j(z))|^{\omega(j,z)}\right)}{(\|f(z)\||\min\{K_1,\cdots,K_q\}|)^{\sum_{j\in Q}\omega(j,z)}}$$

由式 (2) 可得

$$\prod_{j\in S}\left(\frac{1}{|K_j|^2}\right)^{\omega(j,z)}\leqslant\prod_{t_j\in R^0}\left(\frac{\|f(z)\||K_{t_j}|}{|(f(z+jc),H_{t_j}(z))|}\right)\frac{\left(\prod_{t_j\in R}|(f(z+jc),H_{t_j}(z))|^{\omega(t_j,z)}\right)\left(\prod_{j\in S}|(f(z),H_j(z))|^{\omega(j,z)}\right)}{(\|f(z)\||\min\{K_1,\cdots,K_q\}|)^{\sum_{j\in Q}\omega(j,z)}}$$

由引理 1 的性质 b 可得

$$\prod_{j\in S}\left(\frac{1}{|K_j|^2}\right)^{\omega(j,z)}\leqslant\prod_{t_j\in R^0}\left(\frac{\|f(z)\||K_{t_j}|}{|(f(z+jc),H_{t_j}(z))|}\right)\frac{\left(\prod_{t_j\in R}|(f(z+jc),H_{t_j}(z))|^{\omega(t_j,z)}\right)\left(\prod_{j\in S}|(f(z),H_j(z))|^{\omega(j,z)}\right)}{(\|f(z)\||\min\{K_1,\cdots,K_q\}|)^{\tilde{\omega}(z)(q-2N+n-1)+n+1}}\leqslant$$
$$\left(\frac{\prod_{t_j\in R^0}|K_{t_j}|}{\left|\prod_{i=0}^{n}(f(z+ic),H_{t_i}(z))\right|}\right)\frac{\left(\prod_{t_j\in R}|(f(z+jc),H_{t_j}(z))|^{\omega(t_j,z)}\right)\left(\prod_{j\in S}|(f(z),H_j(z))|^{\omega(j,z)}\right)}{(\|f(z)\||\min\{K_1,\cdots,K_q\}|)^{\tilde{\omega}(z)(q-2N+n-1)}}$$

移项可得

$$\|f(z)\|^{\tilde{\omega}(z)(q-2N+n-1)}\leqslant\frac{\left(\prod_{t_j\in R^0}|K_{t_j}|\right)\left(\prod_{j\in S}|K_j|^{2\omega(j,z)}\right)}{|\min\{K_1,\cdots,K_q\}|^{\tilde{\omega}(z)(q-2N+n-1)}}\frac{\left(\prod_{t_j\in R}|(f(z+jc),H_{t_j}(z))|^{\omega(j,z)}\right)\left(\prod_{j\in S}|(f(z),H_j(z))|^{\omega(j,z)}\right)}{\left|\prod_{i=0}^{n}(f(z+ic),H_{t_i}(z))\right|}$$

取适当的K，且K满足：

$$\frac{\left(\prod_{t_j\in R^0}\left|K_{t_j}\right|\right)\cdot\left(\prod_{j\in S}\left|K_j\right|^{2\omega(j,z)}\right)}{\left|\min\{K_1,\cdots,K_q\}\right|^{\tilde{\omega}(z)(q-2N+n-1)}}\leqslant\frac{\left(\prod_{t_j\in R^0}\left|K_{t_j}\right|\right)\cdot\left(\prod_{j\in S}\left|K_j\right|^{2\left(\frac{n+1}{N+1}\right)}\right)}{\left|\min\{K_1,\cdots,K_q\}\right|^{\left(\frac{n+1}{2N-n+1}\right)(q-2N+n-1)}}=K$$

K取值与j有关。

引理 2 得证。

引理 3[3, 7]　设$f:\mathbb{C}\to\mathbb{P}^n(\mathbb{C})$为全纯曲线，$\tilde{f}=(f_0,f_1,\cdots,f_n)$为$f$的既约表示，则$W^{\Delta_c}(f)\neq 0$当且仅当$f$在$\mathcal{P}_c$中线性非退化。

引理 4[6]　$T:[0,+\infty)\to[0,+\infty)$为单调递增的连续函数，$s\in(0,+\infty)$，$\limsup\limits_{r\to\infty}\dfrac{\log\log T(r)}{\log r}=\varsigma<1$，$\delta\in(0,1-\varsigma)$，则有

$$T(r+s)=T(r)+o\left(\frac{T(r)}{r^{\delta}}\right)$$

其中，$r\to\infty$，$r\notin E$，$\int_E\frac{\mathrm{d}t}{t}<\infty$。

引理 5[7-9]　设$f(z)$为$\mathbb{C}$中非常值亚纯函数且$f(0)\neq 0$，∞，$\varsigma_2(f)=\varsigma<1$，则有

$$m\left(r,\frac{f(z+c)}{f(z)}\right)=\int_0^{2\pi}\log^+\left|\frac{f(z+c)}{f(z)}\right|\frac{\mathrm{d}\theta}{2\pi}=o\left(\frac{T_f(r)}{r^{1-\varsigma-\varepsilon}}\right)$$

$$m\left(r,\frac{f(z)}{f(z+c)}\right)=\int_0^{2\pi}\log^+\left|\frac{f(z)}{f(z+c)}\right|\frac{\mathrm{d}\theta}{2\pi}=o\left(\frac{T_f(r)}{r^{1-\varsigma-\varepsilon}}\right)$$

$\varepsilon>0$，且ε任意小。

4　定理 1 的证明

现证明定理 1。

证明　$q>2N-n+1$，$Q=\{1,2,\cdots,q\}$，对集合R^0，R，S和Q用下面的元素来表示：
$j\in Q=\{1,2,\cdots,q\}=\{t_0,t_1,\cdots,t_q\}$，$R^0=\{t_0,\cdots,t_n\}\subset R=\{t_0,\cdots,t_n,t_{n+1},\cdots t_N\}\subset Q\backslash S$，$\#R^0=d(R^0)=d(R)$

当$r>0$时，对引理 2 的结论两边同取对数，有

$$\tilde{\omega}(z)(q-2N+n-1)\log\|f(z)\|\leqslant\sum_{t_j\in R}\omega(t_j,z)\log\left|(f(z+jc),H_{t_j}(z))\right|+$$
$$\sum_{j\in S}\omega(j,z)\log\left|(f(z),H_j(z))\right|-\log\left|\prod_{i=0}^{n}(f(z+ic),H_{t_i}(z))\right|+O(1) \tag{3}$$

因为，f在$\mathcal{P}_c$中线性非退化，由引理 3 可知，$W^{\Delta_c}(f)\neq 0$，又因为，$H_j(z)(j\in Q)$在$\mathbb{P}^n(\mathbb{C})$中处于 N-subgeneral 位置，因此，存在一个非奇异矩阵$\boldsymbol{B}(z)$（$\boldsymbol{B}(z)$与$H_j(z)$相关），由文献[10]可得

$$W^{\Delta_c}((f,H_{t_j}(z)),t_j\in R^0)=W^{\Delta_c}(f)\det\boldsymbol{B}(z)$$

所以，式 (3) 可整理为

$$\tilde{\omega}(z)(q-2N+n-1)\log\|f(z)\|\leqslant\sum_{t_j\in R}\omega(t_j,z)\log\left|(f(z+jc),H_{t_j}(z))\right|+$$
$$\sum_{j\in S}\omega(j,z)\log\left|(f(z),H_j(z))\right|+\log^+\frac{W^{\Delta_c}((f,H_{t_j}(z)),t_j\in R^0)}{\left|\prod_{i=0}^{n}(f(z+ic),H_{t_i}(z))\right|}-\log\left|W^{\Delta_c}(f)\det\boldsymbol{B}(z)\right|+O(1)$$

由$\tilde{\omega}(z)>0$，引理 1 的性质 a，c，有

$$(q-2N+n-1)\log\|f(z)\|\leqslant\sum_{t_j\in R}\frac{\omega(t_j,z)}{\tilde{\omega}(z)}\log\left|(f(z+jc),H_{t_j}(z))\right|+\sum_{j\in S}\frac{\omega(j,z)}{\tilde{\omega}(z)}\log\left|(f(z),H_j(z))\right|-\frac{\log\left|W^{\Delta_c}(f)\right|}{\tilde{\omega}(z)}-$$
$$\frac{\log|\det\boldsymbol{B}(z)|}{\tilde{\omega}(z)}+\frac{\log^+\dfrac{W^{\Delta_c}((f,H_{t_j}(z)),t_j\in R^0)}{\left|\prod_{i=0}^{n}(f(z+ic),H_{t_i}(z))\right|}}{\tilde{\omega}(z)}+O(1)\leqslant\sum_{t_j\in R}\log\left|(f(z+jc),H_{t_j}(z))\right|+\sum_{j\in S}\log\left|(f(z),H_j(z))\right|-$$
$$\frac{N+1}{n+1}\log\left|W^{\Delta_c}(f)\right|-\frac{N+1}{n+1}\log|\det\boldsymbol{B}(z)|+\frac{2N-n+1}{n+1}\log^+\frac{W^{\Delta_c}((f,H_{t_j}(z)),t_j\in R^0)}{\left|\prod_{i=0}^{n}(f(z+ic),H_{t_i}(z))\right|}+O(1)$$

两边同时积分，可得

$$(q-2N+n-1)\int_0^{2\pi}\log\left\|f(re^{i\theta})\right\|\frac{d\theta}{2\pi}\leqslant\sum_{t_j\in R}\int_0^{2\pi}\log\left|(f(re^{i\theta}+jc),H_{t_j}(re^{i\theta}))\right|\frac{d\theta}{2\pi}+$$
$$\sum_{j\in S}\int_0^{2\pi}\log\left|(f(re^{i\theta}),H_j(re^{i\theta}))\right|\frac{d\theta}{2\pi}-\frac{N+1}{n+1}\int_0^{2\pi}\log\left|W^{\Delta_c}(f)\right|\frac{d\theta}{2\pi}-\frac{N+1}{n+1}\int_0^{2\pi}\log\left|\det\boldsymbol{B}(re^{i\theta})\right|\frac{d\theta}{2\pi}+$$
$$\frac{2N-n+1}{n+1}\int_0^{2\pi}\log^+\frac{W^{\Delta_c}((f,H_{t_j}(re^{i\theta})),t_j\in R^0)}{\left|\prod_{i=0}^{n}((f(re^{i\theta}+ic),H_{t_i}(re^{i\theta}))\right|}\frac{d\theta}{2\pi}+O(1)$$

已知

$$|\boldsymbol{B}(z)|=\begin{vmatrix}a_{00}(z) & \dots & a_{0n}(z)\\ \vdots & & \vdots\\ a_{n0}(z) & \dots & a_{nn}(z)\end{vmatrix}$$

$$T(r,a_{ji}(z))=o(T_f(r)),\ i,j=0,1,\cdots,n$$

由特征函数的定义可得

$$\int_0^{2\pi}\log\left|\det\boldsymbol{B}(re^{i\theta})\right|\frac{d\theta}{2\pi}=o(T_f(r))$$

于是

$$(q-2N+n-1)\int_0^{2\pi}\log\left\|f(re^{i\theta})\right\|\frac{d\theta}{2\pi}\leqslant\sum_{t_j\in R}\int_0^{2\pi}\log\left|(f(re^{i\theta}+jc),H_{t_j}(re^{i\theta}))\right|\frac{d\theta}{2\pi}+$$
$$\sum_{j\in S}\int_0^{2\pi}\log\left|(f(re^{i\theta}),H_j(re^{i\theta}))\right|\frac{d\theta}{2\pi}-\frac{N+1}{n+1}\int_0^{2\pi}\log\left|W^{\Delta_c}(f)\right|\frac{d\theta}{2\pi}+$$
$$\frac{2N-n+1}{n+1}\int_0^{2\pi}\log^+\frac{W^{\Delta_c}((f,H_{t_j}(re^{i\theta})),t_j\in R^0)}{\left|\prod_{i=0}^{n}((f(re^{i\theta}+ic),H_{t_i}(re^{i\theta}))\right|}\frac{d\theta}{2\pi}+o(T_f(r)) \quad (4)$$

由特征函数定义和定义 1 可得

$$T_f(r)=\int_0^{2\pi}\log\left\|f(re^{i\theta})\right\|\frac{d\theta}{2\pi}+O(1) \quad (5)$$

$$N_{f(z)}(r,H_j(z))=\int_0^{2\pi}\log\left|(f(re^{i\theta}),H_j(re^{i\theta}))\right|\frac{d\theta}{2\pi}+O(1) \quad (6)$$

所以，式(4)可整理为

$$(q-2N+n-1)T_f(r)\leqslant\sum_{t_j\in R}N_{f(z+jc)}(r,H_{t_j}(z))+\sum_{j\in S}N_{f(z)}(r,H_j(z))-\frac{N+1}{n+1}N(r,W^{\Delta_c}(f)=0)+$$
$$\frac{2N-n+1}{n+1}\int_0^{2\pi}\log^+\frac{W^{\Delta_c}((f,H_{t_j}(re^{i\theta})),t_j\in R^0)}{\left|\prod_{i=0}^{n}((f(re^{i\theta}+ic),H_{t_i}(re^{i\theta}))\right|}\frac{d\theta}{2\pi}+o(T_f(r))$$

又

$$N_{f(z+jc)}(r,H_{t_j}(z))=N_{f(z+jc)}(r,H_{t_j}(z+jc))\leqslant N_{f(z)}(r+jc,H_{t_j}(z))$$

因此，可得

$$(q-2N+n-1)T_f(r)\leqslant\sum_{t_j\in R}N_{f(z)}(r+jc,H_{t_j}(z))+\sum_{j\in S}N_{f(z)}(r,H_j(z))-\frac{N+1}{n+1}N(r,W^{\Delta_c}(f)=0)+$$
$$\frac{2N-n+1}{n+1}\int_0^{2\pi}\log^+\frac{W^{\Delta_c}((f,H_{t_j}(re^{i\theta})),t_j\in R^0)}{\left|\prod_{i=0}^{n}((f(re^{i\theta}+ic),H_{t_i}(re^{i\theta}))\right|}\frac{d\theta}{2\pi}+o(T_f(r))$$

由式 (3) 和式 (4) 易知，$N_f(r,H_{t_j}(z))\leqslant T_f(r)$，所以，

$$\lambda_{t_j}=\limsup_{r\to\infty}\frac{\log^+\log^+N_f(r,H_{t_j}(z))}{\log r}\leqslant\varsigma_2(f)=\varsigma<1$$

由引理 4 可得

$$N_f(r+t_jc,H_{t_j}(z))\leqslant N_f(r,H_{t_j}(z))+o\left(\frac{T_f(r)}{r^{1-\varsigma-\varepsilon}}\right)$$

所以，上式可整理为

$$(q-2N+n-1)T_f(r)\leqslant\sum_{j\in R}N_f(r,H_j(z))+\sum_{j\in S}N_f(r,H_j(z))-\frac{N+1}{n+1}N(r,W^{\Delta_c}(f)=0)+$$
$$\frac{2N-n+1}{n+1}\int_0^{2\pi}\log^+\frac{W^{\Delta_c}((f,H_{t_j}(re^{i\theta})),t_j\in R^0)}{\left|\prod_{i=0}^{n}(f(re^{i\theta}+ic),H_{t_i}(re^{i\theta}))\right|}\frac{d\theta}{2\pi}+o(T_f(r))+o\left(\frac{T_f(r)}{r^{1-\varsigma-\varepsilon}}\right)\leqslant\sum_{j\in Q}N_f(r,H_j(z))-$$
$$\frac{N+1}{n+1}N(r,W^{\Delta_c}(f)=0)+\frac{2N-n+1}{n+1}\int_0^{2\pi}\log^+\frac{W^{\Delta_c}((f,H_{t_j}(re^{i\theta})),t_j\in R^0)}{\left|\prod_{i=0}^{n}(f(re^{i\theta}+ic),H_{t_i}(re^{i\theta}))\right|}\frac{d\theta}{2\pi}+o(T_f(r))+o\left(\frac{T_f(r)}{r^{1-\varsigma-\varepsilon}}\right) \quad (7)$$

记 $g_{t_j}(z+jc)=(f(z+jc),H_{t_j}(z+jc))=(f(z+jc),H_{t_j}(z)),t_j\in R^0$ ，有

$$\frac{W^{\Delta_c}((f(z),H_{t_j}(z)),t_j\in R^0)}{\left|\prod_{i=0}^{n}(f(z+ic),H_{t_i}(z))\right|}=\frac{\begin{vmatrix} g_{t_0}(z) & g_{t_1}(z) & \cdots & g_{t_n}(z)\\ g_{t_0}(z+c) & g_{t_1}(z+c) & \cdots & g_{t_n}(z+c)\\ \vdots & \vdots & & \vdots\\ g_{t_n}(z+nc) & g_{t_1}(z+nc) & \cdots & g_{t_n}(z+nc)\end{vmatrix}}{\left|g_{t_0}(z)g_{t_1}(z+c)\ldots g_{t_n}(z+nc)\right|}=$$

$$\frac{\begin{vmatrix} 1 & \dfrac{g_{t_1}(z)}{g_{t_0}(z)} & \cdots & \dfrac{g_{t_n}(z)}{g_{t_0}(z)}\\ 1 & \dfrac{g_{t_1}(z+c)}{g_{t_0}(z+c)} & \cdots & \dfrac{g_{t_n}(z+c)}{g_{t_0}(z+c)}\\ \vdots & \vdots & & \vdots\\ 1 & \dfrac{g_{t_1}(z+nc)}{g_{t_0}(z+nc)} & \cdots & \dfrac{g_{t_n}(z+nc)}{g_{t_0}(z+nc)}\end{vmatrix}}{\left|\dfrac{g_{t_1}(z+c)}{g_{t_0}(z+c)}\cdots\dfrac{g_{t_n}(z+nc)}{g_{t_0}(z+nc)}\right|}=\frac{\begin{vmatrix} 1 & 1 & \cdots & 1\\ 1 & \left(\dfrac{g_{t_1}(z+c)}{g_{t_0}(z+c)}\right)\Big/\left(\dfrac{g_{t_1}(z)}{g_{t_0}(z)}\right) & \cdots & \left(\dfrac{g_{t_n}(z+c)}{g_{t_0}(z+c)}\right)\Big/\left(\dfrac{g_{t_n}(z)}{g_{t_0}(z)}\right)\\ \vdots & \vdots & & \vdots\\ 1 & \left(\dfrac{g_{t_1}(z+nc)}{g_{t_0}(z+nc)}\right)\Big/\left(\dfrac{g_{t_1}(z)}{g_{t_0}(z)}\right) & \cdots & \left(\dfrac{g_{t_n}(z+nc)}{g_{t_0}(z+nc)}\right)\Big/\left(\dfrac{g_{t_n}(z)}{g_{t_0}(z)}\right)\end{vmatrix}}{\left|\left(\dfrac{g_{t_1}(z+c)}{g_{t_0}(z+c)}\right)\Big/\left(\dfrac{g_{t_1}(z)}{g_{t_0}(z)}\right)\cdots\left(\dfrac{g_{t_n}(z+nc)}{g_{t_0}(z+nc)}\right)\Big/\left(\dfrac{g_{t_n}(z)}{g_{t_0}(z)}\right)\right|}$$

记

$$h_{t_j}(z)=\frac{g_{t_j}(z)}{g_{t_0}(z)}=\frac{(f(z),H_{t_j}(z))}{(f(z),H_{t_0}(z))}$$

则

$$T_{h_{t_j}(z)}(z)\leqslant T_f(r)+O(1)$$

于是，$\varsigma_2(h_{t_j}(z))\leqslant\varsigma_2(f)=\varsigma<1$，因此，由引理 5，有

$$\int_0^{2\pi}\log^+\frac{W^{\Delta_c}((f,H_{t_j}(re^{i\theta})),t_j\in R^0)}{\left|\prod_{i=0}^{n}(f(re^{i\theta}+ic),H_{t_i}(re^{i\theta}))\right|}\frac{d\theta}{2\pi}=o\left(\frac{T_f(r)}{r^{1-\varsigma-\varepsilon}}\right)$$

因此，式(7)可整理为

$$(q-2N+n-1)T_f(r)\leqslant\sum_{j\in Q}N_f(r,H_j(z))-\frac{N+1}{n+1}N(r,W^{\Delta_c}(f)=0)+o(T_f(r))+o\left(\frac{T_f(r)}{r^{1-\varsigma-\varepsilon}}\right)$$

其中，$r>0,r\notin E,\int_E\frac{dt}{t}<\infty$。

定理 1 得证。

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